THE BHÀSVATI ASTRONOMICAL HANDBOOK OF ŚATĀNANDA

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Abstract: The eleventh century Indian astronomer and mathematician Śatānandācārya wrote the Bhàsvatì on CE 7 April 1099. Correspondingly this was the Pournima (Full Moon day) of the first lunar month Caitra of the gata-käli (elapsed kali era) year 4200. This text was a significant contribution to the world of astronomy and mathematics. Śatānanda had adopted the centesimal system for the calculation of the positions and motions of the heavenly bodies, which is similar to the present-day decimal system. His treatise received recognition in the text of the Karana (handbook) grantha. Commentaries of this work were made by different people at different times in history.

Although the Bhàsvatì was reissued about once every century and was well known throughout India, and even abroad, at present it is completely lost and no references to it are available in current works. The main aim of this paper is to outline its contents and bring these to the notice of a wider audience, and to highlight the genius of Śatānanda and his contribution to the world of astronomy and mathematics.

Keywords: Decimal System, Centesimal System, the Bhàsvatì, Tīkās (commentaries), Śatār śa, Dhruvánka (longitude), Ayanār śa

1 INTRODUCTION

The history of development of mathematics in India is as old as the Vedas. From prehistoric times, mathematics began with the rudiments of metrology and computation, of which some fragmentary evidence has survived. The sacred literature of the Vedic Hindus—the Śarhitīs, the Kalpas and the Vedāngas—contains enough information to prove the mathematical abilities of those pioneers who developed this class of literature. Those pioneers, mostly astronomers, used mathematics as an instrument for the calculation of the positions of the stars and the planets. Rather, one can say that such calculations (astronomy) was urged by the development of mathematics (i.e. addition, subtraction, multiplication and division, and also fractions). The division of days, months and the seasons inspired the idea of fractions.

In all ancient calculations the astronomers assigned 360 arīśa (degrees) to a cycle, since 360 is the smallest number divisible by the integers 1 to 10, excluding 7. This trend is still implemented in present-day calculations. However, late in the eleventh century an astronomer named Śatānanda was born in Odisha, and he was successful in developing the mathematical research that was ongoing at this time. For convenience, he converted all cyclic calculations into multiples of one hundred. He used 1200 arīśa while calculating the positions and motions of the planets with respect to the 12 Indian constellations, and he used 2700 arīśa while calculating the positions and motions of the Sun and the Moon with respect to the 27 Nakṣatras.

Śatānanda’s Bhàsvatì introduces very simple methods to calculate celestial parameters, without using trigonometric functions. Therefore it was appreciated by the public, and it spread throughout north India, even though astronomers like Śāmanta Candra Śekhara considered that the calculations were approximate (Ray, 1899). The transformation of arīśa into śatār śa (multiples of hundred) in the Bhàsvatì was Śatānanda’s greatest achievement. Professor Dikshit claims that this mathematical calculation was the initial form of the modern-day decimal system (Dahala, 2012; Vaidya, 1981). Commentaries of Śatānanda’s work were made almost every century during the history of India, but in present-day research the Bhàsvatì is completely ignored by Indian mathematicians and astronomers. Thus, Śatānanda’s pioneering work is little known, even in Odisha.

In this paper we explain the mathematical calculations where Śatānanda has introduced (i) centesimal fractions, and (ii) converted the arīśa (degrees) into śatār śa (multiple of one hundred). Below, in Section 2 we provide biographical details of Śatānanda, while Section 3 contains comments and commentaries on the Bhàsvatì. In Section 4 we explain the mathematics that Śatānanda introduces in the Bhàsvatì, while Section 5 has concluding remarks, including future plans.

2 ŚATĀNANDA: A BIOGRAPHICAL SKETCH

Śatānanda was born in CE 1068 in Puruṣottam-dhāma Purī (Jagannātha Purī), Odisha. From the history of Odisha we know that he may have been a courtier during the Keśari Dynasty (CE 474–1132). During that period, many constructive works were done, the kingdom was peaceful, and patronage was given to scientists and
architects. The state capital, Cuttack, was founded at this time as were the stone embankment along the Kâthajodi River and the Aghanarâlâ bridge of Srikhetra Purî (Acharya, 1879). Śaṭānanda’s Bhâsvatî was the greatest literary achievement of the Keśari Dynasty.

Śaṭānanda wrote his text, which was a guideline to make Paṇcâṅgas (calendars) for the benefit of performing rituals in the Jagannâtha temple in Purî. Since Paṇcâṅgas have an important role in Hindu society Śaṭānanda made accurate calculations of the positions and motions of the heavenly bodies. Hence, there was a saying in Varanasi (which was then the ‘knowledge center’ of India)—“प्रहलेखा भस्वति धन्या” (“Bhâsvatî is the best book to predetermine eclipses”). It is also enlightening to know that the great Indian Hindi poet Mallik Muhammad Jayasi praised Bhâsvatî in his book (see Mishra, 1985):

भस्वती जै बालकन पितृशं पद पूर्ण।
देव मे संवाद करि जनु लोगे हिंस वाह।

This shows Bhâsvatî’s popularity in Indian society.

3  COMMENTARIES ON THE BHÂSVATÎ

There is a commentary on the Bhâsvatî written in Śaka 1417 by Aniruddha of Varanasi, and from this it would appear that there were many other commentaries that had been written about it earlier (see Vaidya, 1981: 110–112). Mâdhava, a resident of Kânauja (Kânyakubja), wrote a commentary on the Bhâsvatî in Śaka 1442. Another commentary on this text was written in Śaka 1607 by Gaṅgâdhara, while the author of a commentary written earlier, in Śaka 1577, is not known. According to the Colebrooke, a commentary written by Balabhadrâ, who was born in the Jumla region of Nepal, was written in Śaka 1330 (Vaidya, op cit.). From the Catalogue of Sanskrit books prepared by Aufrecht, the title of this commentary appears to be Bâlabodhinî. This book was the first mathematics text book in Nepal (Jha et. al., 2006), since mathematical operations like additions, subtractions, multiplications and divisions are explained explicitly in the Bhâsvatî. According to Aufrecht’s Catalogue there are also commentaries on the following texts: the Bhâsvatikarana: Bhâsvatikarana apadhatih; Tattiavprakâśikâ by Râmakrsna, the Bhâsvatikârkaraśmyudâharana by Râmakrsna, the Udâharana by Śaṭānanda and the Udâharaṇa by Vrmdâvana. Similarly, there are commentaries by Achutabhâta, Gopâla, Cakravipradâsa, Râmesvara and Sadânanda, and a Prakrit commentary by Vanamâli. Very recently it was found that there was a commentary of this scripture with examples in the Odia language by Devâda, composed in Śaka 1372, and this is now preserved in the Odisha State Museum in Bhubaneswar. This is a well-explained book on mathematics and heavenly phenomena calculated in the Bhâsvatî. The equinox of 22 March in the year CE 79 in the Gregorian calendar is designated by day 1 of month Caitra of year 1 in the Śaka era. Therefore, 78 years have to be added to the Śaka era to convert it to a Gregorian year (Rao, 2008: 108–114).

As might be expected, most of these commentators hailed from Northern India. When he wrote his masterly History of Indian Astronomy in 1896, Sankar Balakrishna Dikshit regretted that the Bhâsvatî was not known and that there were no references to it in any recently published research (Vaidya, 1981; cf. Dahala, 2012).

Dash (2007: 141–144) advises that copies of these commentaries are presently available in the following libraries:

- Alwar (Rajasthan)
- Asiatic Society, Bengal (Kolkata)
- India Office Library (London)
- Rajasthan Oriental Research Institute (Jodhpur)
- Saraswatibhavan Library (Banaras)
- Visveswarananda Institute (Hoshiarpur)
- Bhandarkar Oriental Research Institute (Pune)

4  THE CONTENTS OF THE BHÂSVATÎ

The Bhâsvatî contains 128 verses in eight Adhikâras (chapters)—see Mishra, 1985). These are:

- Tithyâdhdhruvâdhikâra (Tithi Dhruka)
- Grâhâdhdhruvâdhikâra (Graha Dhruka)
- Paṇcâṅgaspaṣṭâdhikâra (Calculation of Calendar)
- Grahaspaṣṭâdhikâra (True place of Planets)
- Triprasnaâdhikâra (Three problems: Time, Place and Direction)
- Chandragrahaânadhindhikâra (Lunar Eclipse)
- Sûryagrahaânadhindhikâra (Solar Eclipse)
- Parilekhaâdhikâra (Sketch or graphical presentations of eclipses)

In the first śloka of his scripture Śaṭânanda acknowledges the observational work of Varâhamihira which he has used in his calculations. He also claims that his calculations are as accurate as those in the Sûryasiddhânta even though the methods of calculation are completely different. The śloka is as follows:

अष्ट त्यस्य महिषोपदेशार्थसूयसिद्धान्तसम समासत:।

Indian astronomers have differed in their opinions of the rates of precession during different periods with respect to the ‘zero year’. The accumulated amount of precession starting from ‘zero year’ is called ayanârâśa.

There are different methods of calculating the
exact amount of ayanaṃśa:

(i) The Siddhāntas (treatises) furnish the rate for computing it, which is in principle the same as the method of finding the longitude of a star at any given date by applying the amount of precession to its longitude, at some other day.

(ii) Defining the initial point with the help of other data, such as the recorded longitudes of the stars, their present longitudes from the equinox point may be ascertained.

(iii) Knowing the exact year when the initial point was fixed, its present longitude, ayanaṃśa, may be calculated from the known rate of precession.

However it so happens that the results obtained by these three methods do not agree. Śatānanda has his own method of calculation, which was very simple but was considered to be approximate.

The Bhāsvatī has assumed Śa ka 450 (CE 528) as the zero precession year and 1° as the rate of precession per year. However in his 61-page introduction to the Siddhānta Darpana Joghesh Chandra Roy claims that the zero precession year adopted in the Bhāsvatī is Śa ka 427 (i.e. CE 505). He arrived at this number by making the reverse calculation. The calculation of ayanaṃśa (precession) is explained in first śloka of the fifth chapter, Triprasāṇadhikāra:

शक्रनांकत्तां चक्रांलिहितां च चक्कांश्चलोधे च चक्रांतीकांशः सु।।
अर्धां गाः तेडदुःस्या कुच्चे न्दृश्वंधुः दुःश्वंधुः प्रमाणेः॥

The meaning of this śloka is: subtract 450 from the past years of the Śālvāhāna (Śa ka) and then divide it with 60. The quotient is the ayanaṃśa (precession). Add the ayanaṃśa to the ahargana to bring the proof of day night duration.

Here is an example: If we will subtract 450 from Śa ka 1374, it will be 924. Dividing 924 by 60 becomes 15|24. By adding this value to the ahargana 27 the result becomes the sāyana dinagana as 42|24. The table for ‘zero ayanaṃśa’ year and the annual rate of precession adopted in the different scriptures are given in Table 1 above.

It can be seen from Table 2 that the sidereal periods of the Sun and the Moon calculated in the Bhāsvatī are almost the same as in the Surya Siddhānta and is notable improvement compared to the periods of the other planets, having regard to the comparatively slow motion of Jupiter and Saturn.

From the date he dedicated his Bhāsvatī, Śatānanda very cleverly introduced a new calendar for the benefit of society. Many calendars had been introduced by this time (such as the Śakābda, Gatakali, Hiijirābda and Khrīṣṭābda), and Śatānanda took the Śakābda and Gatakali Calendars as his reference calendar and initialized his Śastrābda Calendar. He explained the method of converting the Śakābda and the Gatakali Calendars into his Śastrābda Calendar in the first chapter (i.e. tīthāyādī-dhruvādhikāra) of the Bhāsvatī. The relevant śloka, and its exact translation, are given below:

तत्कालिष्ठया शत्रुष्टविचिह्ने स्तरकवचकमर्वयंशः
शरको नवाशीलकुशणासुके कपोऽवयस्मद्यक्रमः कुः।।
विहर्व्यस्यंतंव्यंतं शत्रुष्टविचिह्ने कपोऽवयस्यं।
कालिष्ठे स एव॥

Gatakali can be ascertained by adding nava
-9 adri -7 indu -1, kršānu -3, hence 3179 to Śakābda. Subtract viyat -0 nabhā -0 loṣan -2 veda -4, hence 4200 from Gatakali, the result is known as Śastrābda āpinda.

Here is an example. The above method has been implemented to convert the present year CE 2019 to the Śastrābda Calendar. The present year CE 2019 – 1941Śakābda. Śakābda 1941 + 3179 = 5120Gatakali. Gatakali 5120 – 4200 = 920 Śastrābda.

Table 1: Zero Ayanāṃśa Year and Annual Rate of Precession.

<table>
<thead>
<tr>
<th>Sūrya Siddhānta</th>
<th>Annual rate of precession</th>
<th>Zero year of equinox in CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>54°</td>
<td>499</td>
<td></td>
</tr>
<tr>
<td>Soma Siddhānta</td>
<td>54°</td>
<td>499</td>
</tr>
<tr>
<td>Laghu-Vasistha Siddhānta</td>
<td>54°</td>
<td>499</td>
</tr>
<tr>
<td>Grahalaghava</td>
<td>60°</td>
<td>522</td>
</tr>
<tr>
<td>Bhāsvatī</td>
<td>60°</td>
<td>528</td>
</tr>
<tr>
<td>Brhatanṛttitā, Mahā (Quoted by Bhāskara-II)</td>
<td>59.9°</td>
<td>505</td>
</tr>
<tr>
<td>Modern data</td>
<td>50.27</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Sidereal Periods in Mean Solar Days.

<table>
<thead>
<tr>
<th>Planets</th>
<th>European Astronomy</th>
<th>Sūrya Siddhānta</th>
<th>Siddhānta Siromani</th>
<th>Siddhānta Darpana</th>
<th>Bhāsvatī</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>365.25637</td>
<td>365.25875+00238</td>
<td>365.25843+00206</td>
<td>365.25875+00238</td>
<td>365.25865+00228</td>
</tr>
<tr>
<td>Moon</td>
<td>27.32166</td>
<td>27.32167+00001</td>
<td>27.32114+00052</td>
<td>27.32167+00001</td>
<td>27.32160+00006</td>
</tr>
<tr>
<td>Mars</td>
<td>686.9794</td>
<td>686.9975+0181</td>
<td>686.9979+0185</td>
<td>686.9857+0063</td>
<td>686.9692-0102</td>
</tr>
<tr>
<td>Mercury</td>
<td>87.9692</td>
<td>87.9585+0107</td>
<td>87.9699+0007</td>
<td>87.9701+0009</td>
<td>87.9672-0020</td>
</tr>
<tr>
<td>Venus</td>
<td>224.7007</td>
<td>224.6985-0022</td>
<td>224.9679-0028</td>
<td>224.7023+0016</td>
<td>224.7025+0018</td>
</tr>
<tr>
<td>Saturn</td>
<td>10759.2197</td>
<td>10765.7730+6.5533</td>
<td>10765.8152+6.5955</td>
<td>10759.7605+5408</td>
<td>10759.7006+0599</td>
</tr>
</tbody>
</table>
and 920 years have passed. However, in this paper I have referred to the \textit{tikās} made in Šaka 1374 (CE 1452) i.e. Šastrābda 353. Therefore all the examples mentioned here are in Šastrābda 353.

In his chapter \textit{tithyādi-drhuvāṅkāra}, Śatānanda gives the method of determining the solar days (\textit{tithī}) and the longitudes (\textit{drhuvā}) of the nine planets: the Sun (\textit{Rāvi}), the Moon (\textit{soma}), Mars (\textit{Maṅgala}), Mercury (\textit{Budha}), Venus (\textit{Śukra}), Jupiter (\textit{Bṛhaspatī}), Saturn (\textit{Śani}), and Rāhu and Ketu (the ‘shadow planets’).

Śatānanda starts his calculations from the Sun. In this same chapter (Chapter 1), in \\textit{slokas} 4 and 5 he gives an empirical method for determining the longitude (\textit{drhuvāṅkāra}) of the Sun. The \\textit{slokas} are shown below:

\begin{quote}
\textbf{단종략관-수두수량의학적요:}

\textbf{अधि प्रक्षे सिनिदिशयाश्रीमहायमनकम सम्बन्धातः}

\textbf{वायुधात्रिणिः। सत्रायुधात्रिणिःसत्रायुधात्रिणिःनीतिविश्वास ॥ ॥}

\textbf{लक्षण विशेषसंयुक्त सुर्यविशेषसंयुक्त स्वातः।}

\textbf{शेषं हेरे प्रोक्ता पूर्ण ग्रहणा लक्ष रूपेहरिको द्वारा: स्वातः। ॥ ॥}

Multiply svara (7) \\textit{sūrya} (0) dik (10) 1007 to Šastrābda and add tāna (49) agra (3) 349 and divide by aśāṣata (800) add agra (6) to the quotient and divide the quotient by naga (7). The remainder is the sarvāstra-pālaka of \\textit{sūrya}. By subtracting it from the divisor \\textit{Śuddhi} comes. Keep this value in two places. Divide by 108 to the digit of one place. That is the \textit{drhuvā} (longitude) of \textit{Madhyama Sūrya}. The quotient should be taken up to three places.
\end{quote}

Mathematically this can be expressed as:

\begin{align*}
\text{Śastrābda } & 920 \times 1007 = 926440 \\
926440 + 349 = 926789. \\
926789 \div 800 = 1158, \text{ with a reminder of } 389 (1) \\
1158 + 6 = 1164 \div 7 = 166, \text{ with a reminder of } 2 = \text{the second graha (planet)} \\
\text{From Sun, i.e. Maṅgala is the Sarvāstra pālaka} \\
\text{From (1), } 800 \div 389 = 411 \text{ \\textit{Śuddhi}} \\
\text{Śuddhi } 411 + 108 = 3 \text{ arhīśa, with a reminder of 87} \\
87 \times 60 = 5220 \div 108 = 48 \text{ kalā, with a reminder of 36} \\
36 \times 60 = 2160 \div 108 = 20 \text{ vikāla}
\end{align*}

So the \textit{drhuvāṅkāra} (longitude) of the rising Sun on \textit{Caitra Śukla Pūrmīmā} (the Full Moon day of the month of \textit{Caitra}) is 5|43|20 \textit{arhīśa}, or 5 \textit{arhīśa} 43 \textit{kāla} 20 \textit{vikāla}. In the \textit{Bhāṣavatī}, Śatānanda first initialized the position of the planets on \textit{Caitra Śukla Pūrmīmā} and then calculated the rate of motion, position and time taken by the planets to complete one rotation in their orbits from the \textit{aharagna} (the day count), unlike other sidhāṅatas, including the \\textit{Śūryaśiddhānta}, which take the starting point approximately from the date of the beginning of civilization (i.e. 6 \textit{manu} + 7 \textit{Sandhī} + 27 \textit{mahāyuga} + 3 \textit{yuga} + present years elapsed from \textit{kaliyuga}) for this purpose. Therefore, the number is huge, so there is every possibility of making mistakes. Despite these simplifications, the \textit{Bhāṣavatī} was still regarded as an authority for the calculation of eclipses.

### 4.1 The Implementation of \textit{Śatāṁśa}

Ancient Indian astronomers believed that the 12 constellations and 27 \textit{Nakṣatras} affected human life. They took 360 \textit{arhīśa} approximately for one rotation, in 365 days, approximately 1° for one day, and specified 30 \textit{arhīśa} for each constellation, and 40/3 \textit{arhīśa} for each star out of 12 constellations and 27 \textit{Nakṣatras} respectively.

Śatānanda very cleverly multiplied 30/4 by 360 \textit{arhīśa} to make it a multiple of one hundred without losing the generality: 360 \times 30/4 = 2700 \textit{arhīśa}. Hence each constellation has 225 \textit{arhīśa}, and each \textit{nakṣatra} has 100 \textit{arhīśa}. He adopted 2700 \textit{arhīśa} for the calculation of the motions (\textit{Sphuṭagati}) of the Sun, the Moon, Rāhu and Ketu. However he adopted 1200 \textit{arhīśa} for the calculation of the motions of the other planets, Mars, Mercury, Venus Jupiter and Saturn, by taking each constellation as 100 \textit{arhīśa} and 400/9 for each \textit{nakṣatra} to avoid dealing with huge numbers.

In Chapter IV (\textit{Graha spāstādhiṅkāra}), Śatānanda introduces the concept of \textit{śatāṁśa} while determining the positions of the planets. As an example, in \textit{sloka} 4.10 he explains the positions of Rāhu and Ketu as follows:

\begin{quote}
\begin{align*}
\text{राहकुटप्ताविवध: -} \\
\text{अहंकारो देवस्वं दध्यां धर्मध्यायं भवतीर्यातः।} \\
\text{खचन्त्रात्मनस्तिरी मुक्त साचानन्द्युक्त सुप्रत रहृपुषकं।} \\
\text{14. १०।}
\end{align*}
\end{quote}

(Multiply \textit{dinagaṇa} by \textit{veda} – 4 and then divide by \textit{daśa} 10. Add the quotient to the last given \textit{drhuvā} (longitude). Subtract it from \textit{व-0-व-0 अग-7 नें-2}, hence 2700. That is Rāhu. Again by dividing the given number by 225 the \textit{rāśi} (constellation) of Rāhu will come.

Then by adding \textit{cakrārdha} 1350 to Rāhu, Ketu comes. And by dividing the position number of Ketu by 225, \textit{rāśi} (constellation) of \textit{ketu} can be determined.

Mathematically

\begin{align*}
\text{Ahargaṇa } & 27 \times 4 \div 108 \times 10 = 10|48|0 \\
\text{The longitude of } & \text{rāhu (पाठ द्रहुवांका) is calculated from the procedure in } \text{Tithyādhruvāṅkāra} \text{ for the year CE 2019 (Śastrābda 920)} \\
4091|01 + 2 = 2045|01 + 10|48|0 = 2056|31 \\
2700 \div 2056|31 = \text{Rāhu Sphuṭha 643|42} \\
\text{Rāhu 643|42} + 100 \div 6 = \text{with a reminder of 43|42}
\end{align*}

This shows that on \textit{ahargaṇa} 27 \textit{Rāhu} lies in \textit{rāśi} \textit{Mithuna} (Gemini) and \textit{nakṣatra} \textit{Punarbasu}.\textbf{}}
Since the motions of Rāhu and Ketu have to be calculated opposite to the motions of the planets, the cakrārṣha 1350 + Rāhu 64|342 = Ketu 1993|42

Here Satānanda took the cakrārṣha (half rotation) as 1350, as one cakra (rotation) is 2700 arhṣa.

It was known that Rāhu and Ketu points are opposite to each other (180° apart) in a circle and when the Moon is near the Rāhu point then there is a chance of getting lunar eclipse and when is on Ketu point Solar eclipse occurs (see Figure 1).

Ketu 1993|42 ÷ 100 = 19 with reminder 93|42

This shows that Ketu lies on rāṣi Dhanu (Śagittarius) and the Mula Nakṣatra.

Implementation of Satānanda had a significant role in predetermining solar and lunar eclipses. This was because (1) 2700 arhṣa is a very big number in comparison to 360 arhṣa, and (2) assigning 100 arhṣato each nakṣatra or constellation could avoid many errors while taking fractions.

![Figure 1: A schematic diagram (not to scale) showing the relative positions of the Sun, the Earth and the Moon for the calculations of the times of solar and lunar eclipses (diagram: Sudhira Panda).]

### 4.2 Calculating Time According to the Bhāsvatī

In this section we want to show the simplified method introduced in the Bhāsvatī to calculate time from gnomonic shadows.

As an example: calculation of time on 15 June of this year (2019), when the shadow of the 12 unit gnomon becomes 15 units.

**Answer:** Here the equinoxial day is 23 March.

So the number of days elapsed = 8 days of March + 30 days of April + 31 days of May + 15 days of June = 84 days

or 30 days Aries + 30 days Taurus + 24 days Gemini = 84 days

Now to calculate carārdhalitā for the month of Aries = 30+30/2 = 45
for the month of Taurus = 30+30/6 = 35
for the month of Gemini = 24/2 = 12

So carārdhalitā = 45 + 35 + 12 = 92 = danda1|32 līta on the day required

**Dinārda** = 15 +1|32 = 16|32 danda

Now, to calculate madhya prabhā (which is the mid-day Sun’s rays)

Carārdhalitā = 92 × 6 = 552/10 = 55|12

552 – 55|12 = (496|48)/10 = 49|41

On 15 June the Sun is in the northern hemisphere. So the above number should be kept as it is.

Now 49|41 – akṣa 44|43 = 4|58 → madhya prabhā

Here the gnomonic shadow or āṣṭachāyā = 15|0

āḍgula × 10 = 150 + 100 = 250

250 – madhya prabhā 4|58 = 245|02 = 245 × 60

+2 = 14702 → śaṅku

now dinārda = 16|32 = 16 × 60 + 32 = 992

992 × 100 = 99200

99200/14702 = danda 6|45 līta

Now we have to convert this to modern time.

danda 6|45 līta ~ 2 hours and 42 minutes

We know that in Indian astronomy the day starts at sunrise.

**Dinārda** on 15 June is 16|32 ~6 hours and 22 minutes = 6° 22′

Mid-day at 87° longitude is at 12 h – 14 m = 11 h 46 m.

Therefore, 11 h 46 m – 6° 22′ = 5° 24′, which is the time of sunrise.

5° 24′ + 2° 42′ = 8° 06′ is the required time when the shadow of 12 āḍgula śaṅku becomes 15 āḍgula.

### 4.2.1 A Physical Explanation to all the Terms and the Methods Adopted

To know time from the gnomonic shadow there are two terms that are involved in the calculation:

1. Madhyaprabha, and
2. Dinārdaṇḍa

Then, for the calculation of Madhyaprabha and Dinārdaṇḍa we need to calculate carārdha, nāḍī and nāṭa. Nāṭa has two parts, saumyanāta and yamyānāta.

The first step of this method is to decide whether the Sun is in the northern or southern sky. If the Sun is in the north then akṣa has to be subtracted (otherwise it would have to be added). This is because when he wrote the Bhāsvatī, Satānanda had made all his calculations with reference to Puri, Odisha, which is in the northern hemisphere. Therefore, when the Sun travels from the northern to the southern hemisphere it has to pass the equator, the zero equinoxial gnomonic shadow line. Hence, to con-
To know madhya prabhā the carārdhalitā has to be calculated. Multiply 6 with carārdhalitā. Keep the result in two places. Subtract one tenth of it from the number in the second place. If the Sun is in the northern hemisphere then keep the number as it is, otherwise add one third of the number to it. Again divide the number by 10. If the Sun is in southern hemisphere then akṣa has to be added.

Śaṭānanda claimed in the Bhāsvatī that this method of calculation of Carārdha outlined there was entirely his own. According to him, if the Sun is in Aries (Meṣa), then the day count + the half of the day count is the carārdhalitā. If the Sun is in Taurus (Vṛṣa) then Carārdha will be the carārdhalitā of Meṣa + number of days elapsed from Vṛṣa + one sixth of number of days elapsed from Vṛṣa. Again, if the Sun is in Gemini (Mithuna), the half of the days elapsed from the month of Mithuna have to be added to the carārdha of the month Vṛṣa. The result is the carārdhalitā for the month of Gemini (Mithuna). The carārdhalitā for the months of Karkaṭa to Kanyā will decrease in the similar manner, and on Kanyā sarıkṛānti it will be zero. A similar calculation has to be followed if the Sun is in the southern hemisphere.

The half day duration, dinārdha, on Mahāviśvasaṅkrānti is 15 daṇḍa. Calculate the carārdhalitā for the day concerned, add the carārdhalitā to 15 if the Sun is in the northern hemisphere and subtract it if the Sun is in southern hemisphere. The result is the required dinārdha (half day duration) for the day concerned.

Since Śaṭānanda made all his calculations with respect to ahargana, in order to make all of my calculations in some reference frame I adopted the data provided by NASA. The old data table by NASA is given below, where 21 March has been taken as Mahāviśva Sarıkṛānti or Meṣa sarıkṛānti. In the Bhāsvatī, Śaṭānanda mentions that the Sun lies in the northern hemisphere for 187 days and in the southern hemisphere for 178 days, which is the same as in the NASA table.

### Table 3: The mid-day gnomonic shadow on all 12 Sarıkṛānti.

<table>
<thead>
<tr>
<th>Sarıkṛānti Number</th>
<th>Declination of the Sun (ś) in degrees</th>
<th>Right ascension of the Sun (i) in degrees</th>
<th>Midday gnomonic shadow from the modern method</th>
<th>Midday gnomonic shadow from themethod in the Bhāsvatī</th>
<th>Difference and Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>4.3676</td>
<td>4.45</td>
<td>0.0824 = 0.69%</td>
</tr>
<tr>
<td>2</td>
<td>11.5008</td>
<td>30.0</td>
<td>1.7933</td>
<td>1.9788</td>
<td>0.1855 = 1.55%</td>
</tr>
<tr>
<td>3</td>
<td>20.2017</td>
<td>60.0</td>
<td>0.04225</td>
<td>0.0986</td>
<td>0.0557 = 0.46%</td>
</tr>
<tr>
<td>4</td>
<td>23.5</td>
<td>90.0</td>
<td>−0.7339</td>
<td>−0.658</td>
<td>0.0759 = 0.63%</td>
</tr>
<tr>
<td>5</td>
<td>5.20218</td>
<td>120.0</td>
<td>−0.04225</td>
<td>−0.037</td>
<td>0.0795 = 0.68%</td>
</tr>
<tr>
<td>6</td>
<td>11.5003</td>
<td>150.0</td>
<td>1.7933</td>
<td>1.739</td>
<td>−0.543 = 0.45%</td>
</tr>
<tr>
<td>7</td>
<td>0.0</td>
<td>180.0</td>
<td>4.3676</td>
<td>4.45</td>
<td>0.082 = 0.69%</td>
</tr>
<tr>
<td>8</td>
<td>−11.5004</td>
<td>210.0</td>
<td>7.3537</td>
<td>7.496</td>
<td>0.1423 = 1.19%</td>
</tr>
<tr>
<td>9</td>
<td>−20.2017</td>
<td>240.0</td>
<td>10.1414</td>
<td>10.232</td>
<td>0.0906 = 0.73%</td>
</tr>
<tr>
<td>10</td>
<td>−23.5</td>
<td>270.5</td>
<td>11.3875</td>
<td>11.24</td>
<td>−0.1475 = 1.23%</td>
</tr>
<tr>
<td>11</td>
<td>−20.2017</td>
<td>300.0</td>
<td>10.1414</td>
<td>10.148</td>
<td>0.0068 = 0.05%</td>
</tr>
<tr>
<td>12</td>
<td>−11.5008</td>
<td>330.0</td>
<td>7.3537</td>
<td>7.595</td>
<td>0.2413 = 2.025%</td>
</tr>
</tbody>
</table>

consider the gnomonic shadow when the Sun is in southern hemisphere the term akṣa has to be added. According to the Bhāsvatī, the Sun lies in the northern hemisphere, from the vernal equinox to the autumnal equinox, for 187 days (the modern value is 186 days), while it is in the southern hemisphere, from the autumnal equinox to the vernal, for 178 days (the modern value is 179 days).

In the second step we have to calculate carārdha (spreading). As we know, the duration of the day and the night changes every day and is not completely uniform. Therefore to take care of the changes in a day, the duration carārdha has to be calculated. This is an empirical method and Śaṭānanda claims that the method is completely his own and that he did not copy from any previous texts. From Madhyaaprabhā the midday gnomonic shadow for the day concerned can be derived. From the proportion of Madyaaprabhā and Īṣṭachāyā the time can be calculated.

Dinārdhadanda can be calculated by adding carārdhalitā to, or subtracting it from, the dinārdhadanda on Mahāviśvasaṅkrānti (i.e. 15 daṇḍa, depending on whether Sun is in the northern or the southern hemisphere). Table 3 lists the midday gnomonic shadow on all 12 sarıkṛāntis, along with modern data.

The length of the shadow of the gnomon should be recorded at the moment at which the time has been calculated. This is known as Īṣṭachāyā.

$$\text{Īṣṭachāyā} \times 10 + 100 - \text{Madhya prabhā = Ṣaṅku} \tag{2}$$

(Note that this Ṣaṅku is different from the gnomon itself.)

Keep dinārdha (half day duration) of that day. Convert daṇḍa and lītā into lītā by multiplying 60 with daṇḍa and then adding lītā. Now multiply lītā pind with 100 and then divide it by the value of Ṣaṅku in equation (2). The result is the Īṣṭachāyākāla (time). This time is of two types, Gatakāla: from morning up to noon, and Ėsvakāla: from noon through to the evening.
5 CONCLUDING REMARKS

In this paper, the contribution of Śatānanda to the world of mathematics and astronomy has been discussed. Some of the ślokas from his text Bhāsvatī has been translated to explain his achievements. It was necessary in order to prepare an accurate almanac for Hindu society, and mostly for the benefit of the Jagannātha temple at Purusottamadhāma Puri. For this purpose he applied the observational data of Varāhamihira and took CE 450 as the year when the text of the Pañcasiddhāntikā of Varāhamihira was written, as zero ayanāṃśa year. Śatānanda started Sāstrābda from the year he dedicated the Bhāsvatī to society, i.e. CE 7 April 1099 (Mishra 1985). Correspondingly, it was the Pournima (Full Moon day) of the first lunar month Caitra of the gata-kalī (elapsed kali era) year 4200. All calculations in the Bhāsvatī were in Sāstrābda, and he had given rules to convert Sāstrābda to Śakābda and vice versa. Śatānanda has taken the latitude and longitude of Puri in Odisha as his reference point. Maybe it was easy for him to recheck his methods from observations made at his native place.

The most interesting thing found in the Bhāsvatī is that Śatānanda could calculate the position and rate of motion of heavenly bodies quite accurately without using trigonometric functions. Though some ancient astronomers rejected the methodology by saying that was an approximate method, it is interesting to see that this ‘approximate method’ could provide exact solutions when predetermining eclipses. Use of Śatārūṣa (a centesimal system) in the procedure and making a back transform was quite a modern idea that was adopted by Śatānanda. A strong claim exists that the conversion of the sexagesimal system to the centesimal system was the first step that led mathematicians towards the introduction of the decimal system in mathematical calculations (Vaidya, 1981: 110–112). In this context, it is necessary to study the physical and mathematical interpretation of all 128 ślokas in the Bhāsvatī.

A detail study is now in progress to establish the relationship between the method outlined in the Bhāsvatī and the modern European method of predetermining an eclipse.

6 ACKNOWLEDGEMENTS

I am very grateful to Professor Balachandra Rao for helping with the revision of this paper.

7 REFERENCES


8 APPENDIX A: THE METHOD OF CALCULATING THE SIDEREAL PERIOD OF MOON

Step 1. Multiply 90 by ahargana and add Candra Dhruka with it. Divide the result by 2457.
Step 2. Multiply 100 by ahargana and add Kendra dhruka to it. Divide the result by 2756.
Step 3. Divide ahargana by 120 and add the remainder of Step 1. The carārdha of the respective month has to be subtracted from the result. The carārdha for each month is given in Table 4.
Step 4. Divide ahargana by 50 and add the remainder of Step 2. Then divide the result by 100.
Step 5. From the quotient the corresponding Khaṇḍa and Anukhaṇḍa (khaṇḍa + 1) have to taken from Table 5 below. Subtract Khaṇḍa from Anukhaṇḍa, and the result is candra bhoga. The remainder from Step 4 has to be multiplied by candra bhoga. Divide the result by 100. The result has to be added to Khaṇḍa and the result of step 3. The result is candra sphuṭa.

In the similar manner candra sphuṭa for the next day (ahargana) has to be calculated. The positional difference of the day is called candra bhukti (the Moon’s diurnal motion). This motion is not uniform. Therefore for the sidereal calculation I kept on increasing the ahargana until the Moon comes to the same position (candra sphuṭa).

Table 4: The Carardha value that has to be subtracted in different months

<table>
<thead>
<tr>
<th>Name of Sidereal Month</th>
<th>Carārdha</th>
<th>Name of Sidereal Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aries</td>
<td>0</td>
<td>Pisces</td>
</tr>
<tr>
<td>Taurus</td>
<td>1</td>
<td>Aquarius</td>
</tr>
<tr>
<td>Gemini</td>
<td>2</td>
<td>Capricorn</td>
</tr>
<tr>
<td>Cancer</td>
<td>2</td>
<td>Sagittarius</td>
</tr>
<tr>
<td>Leo</td>
<td>1</td>
<td>Scorpio</td>
</tr>
<tr>
<td>Virgo</td>
<td>0</td>
<td>Libra</td>
</tr>
</tbody>
</table>
1. On 21 March the Sun lies on the equator. So we take the Sun’s position at 0° Aries. So the gnomonic shadow will be 4.45. This is little higher than that of modern data (i.e. 4.37 + 0.08)

2. On 21 April, Taurus = 30° = aharga = 31 = 30 +1
carārdhālita = 45 + 1 + 1/6 = 46.17
46.17 × 6 = 277.02 – 27.70 = 249.32/10 = 24.932
madhya prabhā = 44.72 – 24.932 = 19.788
iṣṭachāyā = 19.788/10 = 1.9788

3. On 22 May, Gemini: 60° = aharga = 62 = 30 + 30 + 2
carārdhālita = 45 + 35 + 1 = 81
81 × 6 = 486 – 486/10 = 437.4/10 = 43.74
madhya prabhā = 44.72 – 43.74 = 0.98
iṣṭachāyā = 0.98/10 = 0.098

4. On 22 June, Cancer: 90° = aharga = 93 = 30 + 30 + 3

carārdhālita = 45 + 35 + 33/2 = 96.5
96.5 × 6 = 579 – 57.9 = 521.1/10 = 52.11
madhya prabhā = 44.72 – 52.11 = −7.39
iṣṭachāyā = −7.39/10 = −0.739

5. On 23 July, Leo: 120° = aharga = 124
(In this case there is little change in procedure. It has been mentioned that the Sun lies 187 days in the Northern Hemisphere and 178 days in the Southern Hemisphere. So when aharga exceeds half of the days in a hemisphere then we have to take the smaller part for the carārdhālita calculation. i.e. 187 – 124 = 63. So we have to calculate the carārdhālita of 63 aharga.)
63 = 30 + 30 + 3
carārdhālita = 45 + 35 + 3/2 = 81.5
81.5 × 6 = 501 – 50.1 = 450.9/10 = 45.09
Madhya prabhā = 44.72 – 45.09 = −0.37
iṣṭachāyā = −0.37/10 = −0.037
6. On 22 August, Virgo: $150^\circ = aharga = 154 = 187 - 154 = 33 = 30 + 3$

\begin{align*}
  &carârdhaltâ = 45 + 3 + 3/6 = 48.5 \\
  &48.5 \times 6 = 291 - 29.1 = 261.9/10 = 26.19 \\
  &madhya praabhâ = 44.72 - 26.19 = 18.53 \\
  &istachaya = 18.53/10 = 1.853
\end{align*}

7. On 24 September, Libra: $180^\circ = aharga = 215$

\begin{align*}
  &Shadow length = 4.45
\end{align*}

8. On 22 October, Scorpio: $210^\circ = aharga = 215$

\begin{align*}
  &215 - 187 = (Southern Hemisphere) = 28 \\
  &carârdhaltâ = 28 + 14 = 42 \\
  &\text{(there is little change in procedure for the Southern Hemisphere)} \\
  &42 \times 6 = 252 - 25.2 = 226.8 + 226.8/3 = 302.4/10 = 30.24 \\
  &madhya praabhâ = 44.72 + 30.24 = 74.96 \\
  &istachâyâ = 74.96/10 = 7.496
\end{align*}

9. On 23 November, Sagittarius: $240^\circ = aharga = 247$

\begin{align*}
  &247 - 187 = 60 \\
  &carârdhaltâ = 45 + 35 = 80 \\
  &80 \times 6 = 480 - 48 = 432 + 432/3 = 576/10 = 57.6 \\
  &madhya praabhâ = 44.72 + 57.6 = 102.32 \\
  &istachâyâ = 102.32/10 = 10.232
\end{align*}

10. On 23 December, Capricorn: $270^\circ = aharga = 277$

\begin{align*}
  &277 - 187 = 90 \\
  &\text{Southern Hemisphere} 178 - 90 = 88 \\
  &\text{We have to calculate carârdhaltâ of the smaller part.} \\
  &\text{So carârdhaltâ of 88} = 45 +35 +14 = 94 \\
  &94 \times 6 = 564 - 56.4 = 507.6 + 507.6/3 = 676.8/10 = 67.68 \\
  &madhya praabhâ = 44.72 + 67.68 = 112.4 \\
  &istachâyâ = 112.4/10 = 11.24
\end{align*}

11. On 21 January, Aquarius: $300^\circ = aharga = 306$

\begin{align*}
  &306 - 187 = 119 \\
  &178 - 119 = 59 \\
  &59 = 45 + 29 + 29/6 = 78.83 \\
  &78.83 \times 6 = 473 - 47.3 = 425.7 + 425.7/3 = 567.6/10 = 56.76 \\
  &madhya praabhâ = 44.72 + 56.76 = 101.48 \\
  &istachâyâ = 101.48/10 = 10.148
\end{align*}

12. On 20 February, Pisces: $330^\circ = aharga = 336$

\begin{align*}
  &336 - 187 = 149 \\
  &178 - 149 = 29 \\
  &29 + 29/2 = 43.5 \\
  &43.5 \times 6 = 261 - 26.1 = 234.9 + 234.9/3 = 312.3/10 = 31.23 \\
  &madhya praabhâ = 44.72 + 31.23 = 75.95 \\
  &istachâyâ = 75.95/10 = 7.595
\end{align*}

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