THE *BHĀSVATĪ* ASTRONOMICAL HANDBOOK OF ŚATĀNANDA

Sudhira Panda
Institute of Mathematics and Applications, Andharua,
Bhubaneshwar-751003, Odisha, India.
Email: sudhira@iopb,res,in

Abstract: The eleventh century Indian astronomer and mathematician Śatānandācārya wrote the *Bhāsvatī* on CE 7 April 1099. Correspondingly this was the *Pournima* (Full Moon day) of the first lunar month *Caitra* of the *gata-kali* (elapsed *kali* era) year 4200. This text was a significant contribution to the world of astronomy and mathematics. Śatānanda had adopted the centesimal system for the calculation of the positions and motions of the heavenly bodies, which is similar to the present-day decimal system. His treatise received recognition in the text of the *Karaṇa* (handbook) *grantha*. Commentaries of this work were made by different people at different times in history.

Although the *Bhāsvatī* was reissued about once every century and was well known throughout India, and even abroad, at present it is completely lost and no references to it are available in current works. The main aim of this paper is to outline its contents and bring these to the notice of a wider audience, and to highlight the genius of Śatānanda and his contribution to the world of astronomy and mathematics.

Keywords: Decimal System, Centesimal System, the *Bhāsvatī*, *Ṭīkās* (commentaries), *Śatāṁśa*, *Dhruvāṅka* (longitude), *Ayanāṁśa*

1 INTRODUCTION

The history of development of mathematics in India is as old as the Vedas. From prehistoric times, mathematics began with the rudiments of metrology and computation, of which some fragmentary evidence has survived. The sacred literature of the Vedic Hindus—the *Saṁhītās*, the *Kalpas* and the *Vedāṅgas*—contains enough information to prove the mathematical abilities of those pioneers who developed this class of literature. Those pioneers, mostly astronomers, used mathematics as an instrument for the calculation of the positions of the stars and the planets. Rather, one can say that such calculations (astronomy) was urged by the development of mathematics (i.e. addition, subtraction, multiplication and division, and also fractions). The division of days, months and the seasons inspired the idea of fractions.

In all ancient calculations the astronomers assigned 360 *aṁśa* (degrees) to a cycle, since 360 is the smallest number divisible by the integers 1 to 10, excluding 7. This trend is still implemented in present-day calculations. However, late in the eleventh century an astronomer named Śatānanda was born in Odisha, and he was successful in developing the mathematical research that was ongoing at this time. For convenience, he converted all cyclic calculations into multiples of one hundred. He used 1200 *aṁśa* while calculating the positions and motions of the planets with respect to the 12 Indian constellations, and he used 2700 *aṁśa* while calculating the positions and motions of the Sun and the Moon with respect to the 27 *Nakṣatra*s.

Śatānanda's *Bhāsvatī* introduces very simple methods to calculate celestial parameters, without using trigonometric functions. Therefore it was appreciated by the public, and it spread throughout north India, even though astronomers like Sāmanta Candra Śekhara considered that the calculations were approximate (Ray, 1899). The transformation of *aṁśa* into *śatāṁśa* (multiples of hundred) in the *Bhāsvatī* was Śatānanda's greatest achievement. Professor Dikshit claims that this mathematical calculation was the initial form of the modern-day decimal system (Dahala, 2012; Vaidya, 1981). Commentaries of Śatānanda's work were made almost every century during the history of India, but in present-day research the *Bhāsvatī* is completely ignored by Indian mathematicians and astronomers. Thus, Śatānanda's pioneering work is little known, even in Odisha.

In this paper we explain the mathematical calculations where Śatānanda has introduced (i) centesimal fractions, and (ii) converted the *aṁśa* (degrees) into *śatāṁśa* (multiple of one hundred). Below, in Section 2 we provide biographical details of Śatānanda, while Section 3 contains comments and commentaries on the *Bhāsvatī*. In Section 4 we explain the mathematics that Śatānanda introduces in the *Bhāsvatī*, while Section 5 has concluding remarks, including future plans.

2 ŚATĀNANDA: A BIOGRAPHICAL SKETCH

Śatānanda was born in CE 1068 in Puruṣottamdhāma Puri (Jagannātha Puri), Odisha. From the history of Odisha we know that he may have been a courtier during the Keśari Dynasty (CE 474–1132). During that period, many constructive works were done, the kingdom was peaceful, and patronage was given to scientists and architects. The state capital, Cuttack, was founded at this time as were the stone embankment along the Kāṭhajoḍi River and the Aṭharanalā bridge of Śrikhetra Purī (Acharya, 1879). Śatānanda's *Bhāsvatī* was the greatest literary achievement of the Keśari Dynasty.

Śatānanda wrote his text, which was a guideline to make *Pañcāṅgas* (calendars) for the benefit of performing rituals in the Jagannātha temple in Puri. Since *Pañcāṅgas* have an important role in Hindu society Śatānanda made accurate calculations of the positions and motions of the heavenly bodies. Hence, there was a saying in Varanasi (which was then the 'knowledge center' of India)—"**ग्रहणे भास्वती धन्या**" ("*Bhāsvatī* is the best book to predetermine eclipses"). It is also enlightening to know that the great Indian Hindi poet Mallik Muhammad Jayasi praised *Bhāsvatī* in his book (see Mishra, 1985):

> भास्वती औ व्याकरन पिङ्गल पाठ पुराण।
> वेद मेद सोवात कहि जनु लागे हिय वान ॥

This shows Bhāsvatī's popularity in Indian society.

3 COMMENTARIES ON THE *BHĀSVATĪ*

There is a commentary on the *Bhāsvatī* written in Śaka 1417 by Anirudddha of Varanasi, and from this it would appear that there were many other commentaries that had been written about it earlier (see Vaidya, 1981: 110–112). Mādhava, a resident of Kanauja (Kānyakubja), wrote a commentary on the *Bhāsvatī* in *Śaka* 1442. Another commentary on this text was written in *Śaka* 1607 by Gaṅgādhara, while the author of a commentary written earlier, in *Śaka* 1577, is not known. According to the Colebrooke, a commentary written by Balabhadra, who was born in the Jumula region of Nepal, was written in *Śaka* 1330 (Vaidya, *op cit*.). From the Catalogue of Sanskrit books prepared by Aufrecht, the title of this commentary appears to be *Bālabodhinī.* This book was the first mathematics text book in Nepal (Jha et. al., 2006), since mathematical operations like additions, subtractions, multiplications and divisions are explained explicitly in the *Bhāsvatī*. According to Aufrecht's Catalogue there are also commentaries on the following texts: the *Bhāsvatīkaraṇa: Bhāsvatīkaraṇapaddhati*; *Tattvaprakāśikā* by Rāmakṛṣṇa, the *Bhāsvatīcakraraśmyudāharaṇa* by Rāmakṛṣṇa, the *Udāharaṇa* by Śatānanda and the *Udāharaṇa* by Vṛndāvana. Similarly, there are commentaries by Achutabhaṭṭa, Gopāla, Cakravipradāsa, Rāmeśvara and Sadānanda, and a *Prakrit* commentary by Vanamāli. Very recently it was found that there was a commentary of this scripture with examples in the Odia language by Devīdāsa, composed in *Śaka* 1372, and this is now preserved in the Odisha State Museum in Bhubaneswar. This is a well-explained book on mathematics and heavenly phenomena calculated in the *Bhāsvatī*. The equinox of 22 March in the year CE 79 in the Gregorian calendar is designated by day 1 of month Caitra of year 1 in the *Śaka* era. Therefore, 78 years have to be added to the *Śaka* era to convert it to a Gregorian year (Rao, 2008: 108–114).

As might be expected, most of these commentators hailed from Northern India. When he wrote his masterly *History of Indian Astronomy* in 1896, Sankar Balakrishna Dikshit regretted that the *Bhāsvatī* was not known and that there were no references to it in any recently published research (Vaidya, 1981; cf. Dahala, 2012).

Dash (2007: 141–144) advises that copies of these commentaries are presently available in the following libraries:

- Alwar (Rajasthan)
- Asiatic Society, Bengal (Kolkata)
- India Office Library (London)
- Rajasthan Oriental Research Institute (Jodhpur)
- Saraswatibhavan Library (Banaras)
- Visveswarananda Institute (Hosiarpur)
- Bhandarkar Oriental Research Institute (Pune)

4 THE CONTENTS OF THE *BHĀSVATĪ*

The *Bhāsvatī* contains 128 verses in eight *Adhikāras* (chapters)—see Mishra, 1985). These are:

- *Tīthyādidhruvādhikāra* (*Tithi Dhruva*)
- *Grāhadhruvādhikāra* (*Graha Dhruva*)
- *Pañcāṅgaspaṣṭādhikāra* (Calculation of Calendar)
- *Grahaspaṣṭādhikāra* (True place of Planets)
- *Tripraśnādhikāra* (Three problems: Time, Place and Direction)
- *Chandragrahaṇādhikāra* (Lunar Eclipse)
- *Sūryagrahaṇādhikāra* (Solar Eclipse)
- *Parilekhādhikāra* (Sketch or graphical presentations of eclipses)

In the first *śloka* of his scripture Śatānanda acknowledges the observational work of Varāhamihira which he has used in his calculations. He also claims that his calculations are as accurate as those in the *Sūryasiddhānta* even though the methods of calculation are completely different. The *śloka* is as follows:

> अथ प्रवक्ष्ये मिहिरोपदेशाच्छ्रीसूर्य्यसिद्धान्तसमं समासात्।

Indian astronomers have differed in their opinions of the rates of precession during different periods with respect to the 'zero year'. The accumulated amount of precession starting from 'zero year' is called *ayanāṁśa*.

There are different methods of calculating the

Table 1: Zero *Ayanāṃśa* Year and Annual Rate of Precession.

Siddhānta (treatises)	Annual rate of precession	Zero year of equinox in CE
Sūrya Siddhānta	54"	499
Soma Siddhānta	54"	499
Laghu-*Vasiṣṭha Siddhānta*	54"	499
Grahalāghava	60"	522
Bhāsvatī	60"	528
Bṛhatsaṁhitā, Muñjāla (Quoted by Bhāskara-II)	59.9"	505
Modern data	50.27	

Table 2: Sidereal Periods in Mean Solar Days.

Planets	European Astronomy	*Sūrya Siddhānta*	*Siddhānta Śiromaṇi*	*Siddhānta Darpana*	*Bhāsvatī*
Sun	365.25637	365.25875+00238	365.25843+00206	365.25875+00238	365.25865+00228
Moon	27.32166	27.32167+00001	27.32114–00052	27.32167+00001	27.32160+00006
Mars	686.9794	686.9975+0181	686.9979+0185	686.9857+0063	686.9692–0102
Mercury	87.9692	87.9585+0107	87.9699+0007	87.9701+0009	87.9672–0020
Jupiter	4332.5848	4332.3206–2642	4332.2408–3440	4332.6278+0430	4332.3066–2782
Venus	224.7007	224.6985–0022	224.9679–0028	224.7023+0016	224.7025+0018
Saturn	10759.2197	10765.7730+6.5533	10765.8152+6.5955	10759.7605+5408	10759.7006+0599

exact amount of *ayanāṁśa*:

(i) The *Siddhāntas* (treatises) furnish the rate for computing it, which is in principle the same as the method of finding the longitude of a star at any given date by applying the amount of precession to its longitude, at some other day.

(ii) Defining the initial point with the help of other data, such as the recorded longitudes of the stars, their present longitudes from the equinox point may be ascertained.

(iii) Knowing the exact year when the initial point was fixed, its present longitude, *ayanāṁśa*, may be calculated from the known rate of precession.

However it so happens that the results obtained by these three methods do not agree. Śatānanda has his own method of calculation, which was very simple but was considered to be approximate.

The *Bhāsvatī* has assumed *Śaka* 450 (CE 528) as the zero precession year and 1′ as the rate of precession per year. However in his 61-page introduction to the *Siddhānta Darpaṇa* Jogesh Chandra Roy claims that the zero precession year adopted in the *Bhāsvatī* is *Śaka* 427 (i.e. CE 505). He arrived at this number by making the reverse calculation. The calculation of *ayanāṁśa* (precession) is explained in first *śloka* of the fifth chapter, *Tripraśnādhikāra*:

> शकेन्द्रकालात् खशराब्धिहीनात् षष्ट्याप्तशेषे ह्ययनांशकाः स्युः।
> अहर्गणं तैर्युतमेव कुर्याद् बवेद्द्युवृन्दं द्युनिशोः प्रमाणे॥१॥

The meaning of this *śloka* is: subtract 450 from the past years of the *Śālivāhana* (*Śaka*) and then divide it with 60. The quotient is the *ayanāṁśa* (precession). Add the *ayanāṁśa* to the *ahargaṇa* to bring the proof of day night duration.

Here is an Example: If we will subtract 450 from *Śak*a 1374, it will be 924. Dividing 924 by 60 becomes 15|24. By adding this value to the *ahargaṇa* 27 the result becomes the *sāyana-dinagaṇa* as 42|24. The table for 'zero *ayanāṁśa*' year and the annual rate of precession adopted in the different scriptures are given in Table 1 above.

It can be seen from Table 2 that the sidereal periods of the Sun and the Moon calculated in the *Bhāsvatī* are almost the same as in the *Sūrya Siddhānta* and is notable improvement compared to the periods of the other planets, having regard to the comparatively slow motion of Jupiter and Saturn.

From the date he dedicated his *Bhāsvatī*, Śatānanda very cleverly introduced a new calendar for the benefit of society. Many calendars had been introduced by this time (such as the *Śakābda, Gatakali, Hijirābda* and *Khrīṣṭābda*), and Śatānanda took the *Śakābda* and *Gatakali* Calendars as his reference calendar and initialized his *Śāstrābda Calendar*. He explained the method of converting the *Śakābda* and the *Gatakali Calendars* into his *Śāstrābda Calendar* in the first chapter (i.e. *tithyādi-dhruvādhikāra*) of the *Bhāsvatī*. The relevant *śloka*, and its exact translation, are given below:

> गतकलिः प्रकारान्तरेण शास्त्राब्दविधिश्च-
> शाको नवाद्रीन्दुकृशानुयुक्तः कलेर्भवत्यब्दगणस्तु वृत्तः।
> वियन्नभोलोचनवेदहीनः शास्त्राब्दपिण्डः कथितः स एव॥१.२॥

> *Gatakali* can be ascertained by adding *nava* -9 *adri* -7 *indu* -1, *kṛśānu* -3, hence 3179 to *Śakābda*. Subtract *viyat* -0 *nabhaḥ* -0 *locan* -2 *veda* -4, hence 4200 from *Gatakali*, the result is known as *Śāstrābdapiṇḍa*.

Here is an example. The above method has been implemented to convert the present year CE 2019 to the *Śāstrābda Calendar*. The present year CE 2019 – 78 = 1941*Śakābda*. *Śakābda* 1941 + 3179 = 5120*Gatakali. Gatakali* 5120 – 4200 = 920 *Śāstrābda*. Hence as per the record, the *Bhāsvatī* was written in CE 1099 and 920 years have passed. However, in this paper I have referred to the *ṭīkās* made in *Śaka* 1374 (CE 1452) i.e. *Śāstrābda* 353. Therefore all the examples mentioned here are in *Śāstrābda* 353.

In his chapter *tithyādi-dhruvādhikāra,* Śatānanda gives the method of determining the solar days (*tithi*) and the longitudes (*dhruva*) of the nine planets: the Sun (*Ravi*), the Moon (*soma*), Mars (*Maṅgala*), Mercury (*Budha*), Venus (*Śukra*), Jupiter (*Bṛhaspati*), Saturn (*Śani*), and *Rāhu* and *Ketu* (the 'shadow planets').

Śatānanda starts his calculations from the Sun. In this same chapter (Chapter 1), in *ślokas* 4 and 5 he gives an empirical method for determining the longitude (*dhrūvāṅka*) of Sun. The *ślokas* are shown below:

> *संवत्सरपालक-शुद्धिसूर्य्यध्रुवविधय :-*
>
> अथ प्रवक्ष्ये मिहिरोपदेशाच्छ्रीसूर्य्यसिद्धान्तसमं समासात्।
> शास्त्राब्दपिण्डः स्वरशून्यदिग्घ्नस्तानाग्नियुक्तोष्टशतैर्विभक्तः॥१.४॥
> लब्धन्नगैः शेषितमङ्गयुक्तः सूर्य्यादिसंवत्सरपालकः स्यात्।
> शेषं हरे प्रोज्झ्य पृथग् गजाशा लब्धं रवेरौदयिको ध्रुवः स्यात्॥१. ५॥
>
> Multiply *svara* (7) *śūnya* (0) *dik* (10) 1007 to *Śāstrābda* and add *tāna* (49) *agni* (3) 349 and divide by *aṣṭaśata* (800) add *aṅga* (6) to the quotient and divide the quotient by *naga* (7). The remainder is the *saṁvatsarapālaka* of *Sūrya*. By subtracting it from the divisor *Śuddhi* comes. Keep this value in two places. Divide by 108 to the digit of one place. That is the *dhruva* (longitude) of *Madhyama Sūrya*. The quotient should be taken up to three places.

Mathematically this can be expressed as:

Śāstrābda 920 × 1007 = 926440
926440 + 349 = 926789.
926789 ÷ 800 = 1158, with a reminder of 389 (1)
1158 + 6 = 1164 ÷ 7 = 166, with a reminder of 2 = the second *graha* (planet) from Sun, i.e. *Maṅgala* is the *Saṁvatsara pālaka*
From (1), 800 – reminder 389 = 411 *Śuddhi*
Śuddhi 411 ÷ 108 = 3 *aṁśa*, with a reminder of 87
87 × 60 = 5220 ÷ 108 = 48 *kalā*, with a reminder of 36
36 × 60 = 2160 ÷ 108 = 20 *vikalā*

So the *dhrūvāṅka* (longitude) of the rising Sun on *Caitra Śukla Pūrṇimā* (the Full Moon day of the month of *Caitra*) is 5|43|20 *aṁśa*, or 5 *aṁśa* 43 *kāla* 20 *vikāla*. In the *Bhāsvatī,* Śatānanda first initialized the position of planets on *Caitra Śukla Pūrṇimā* and then calculated the rate of motion, position and time taken by the planets to complete one rotation in their orbits from the *ahargaṇa* (the day count), unlike other *siddhāntas*, including the *Sūryasiddhānta*, which take the starting point approximately from the date of the beginning of civilization (i.e. 6 *manu* + 7 *Sandhi* + 27 *mahāyuga* + 3 *yuga* + present years elapsed from *kaliyuga*) for this purpose. Therefore, the number is huge, so there is every possibility of making mistakes. Despite these simplifications, the *Bhāsvatī* was still regarded as an authority for the calculation of eclipses.

4.1 The Implementation of *Śatāṁśa*

Ancient Indian astronomers believed that the 12 constellations and 27 *Nakṣatras* affected human life. They took 360 *aṁśa* approximately for one rotation, in 365 days, approximately 1° for one day, and specified 30 *aṁśa* for each constellation, and 40/3 *aṁśa* for each star out of 12 constellations and 27 *Nakṣatras* respectively.

Śatānanda very cleverly multiplied 30/4 by 360 *aṁśa* to make it a multiple of one hundred without losing the generality: 360 × 30/4 = 2700 *aṁśa*. Hence each constellation has 225 *aṁśa*, and each *nakṣatra* has 100 *aṁśa*. He adopted 2700 *aṁśa* for the calculation of the motions (*Sphuṭagati*) of the Sun, the Moon, Rāhu and Ketu. However he adopted 1200 *aṁśa* for the calculation of the motions of the other planets, Mars, Mercury, Venus Jupiter and Saturn, by taking each constellation as 100 *aṁśa* and 400/9 for each *Nakṣatra* to avoid dealing with huge numbers.

In Chapter IV (*Graha spaṣṭādhikāra*), Śatānanda introduces the concept of *śatāṁśa* while determining the positions of the planets. As an example, in *śloka* 4.10 he explains the positions of *Rāhu* and *Ketu* as follows:

> राहुकेतुस्पष्टविधि:-
>
> अहर्गणं वेदहतं दशाप्तं ध्रुवार्द्धयुक्तं भवतीह पातः।
> खखागनेत्रान्तरितो मुखं स्याच्चक्रार्द्धयुक्तं स्फुट राहुपुच्छः॥४.१०॥
>
> (Multiply *dinagaṇa* by *veda* – 4 and then divide by *daśa* 10. Add the quotient to the last given *dhruva* (longitude). Subtract it from ख – 0 ख – 0 अग – 7 नेत्र – 2, hence 2700. That is *Rāhu*. Again by dividing the given number by 225 the *rāśi* (constellation) of *Rāhu* will come.

Then by adding *cakrārdha* 1350 to *Rāhu, Ketu* comes. And by dividing the position number of *Ketu* by 225, *rāśi* (constellation) of *ketu* can be determined.

Mathematically

Ahargaṇa 27 × 4 =108 ÷ 10 = 10|48|0
The longitude of *rāhu* (*pāta dhrūvāṅka*) is calculated from the procedure in *Tīthyādidhruvādhikāra* for the year CE 2019 (*Śāstrābda* 920)
4091|01 ÷ 2 = 2045|01 + 10|48|0 = 2056|31
2700 – 2056|31 = *Rāhu Sphuṭa* 643|42
Rāhu 643|42 ÷ 100 = 6 with a reminder of 43|42
This shows that on *ahargaṇa* 27 *Rāhu* lies in *rāśi Mithuna* (Gemini) and *Nakṣatra Punarbasu.*

Since the motions of *Rāhu and Ketu* have to be calculated opposite to the motions of the planets,
the *cakrārdha* 1350 + *Rāhu* 643|42 = *Ketu* 1993|42
Here Śatānanda took the *cakrārdha* (half rotation) as 1350, as one *cakra* (rotation) is 2700 *aṁśa*.
It was known that *Rāhu* and *Ketu* points are opposite to each other (180° apart) in a circle and when the Moon is near the Rāhu point then there is a chance of getting lunar eclipse and when is on Ketu point Solar eclipse occurs (see Figure 1).
Ketu 1993|42 ÷ 100 =19 with reminder 93|42
This shows that *Ketu* lies on *rāśi Dhanu (Sagittarius)* and the *Mula Nakṣatra.*

Implementation of *Śatāṁśa* had a significant role in predetermining solar and lunar eclipses. This was because (1) 2700 *aṁśa* is a very big number in comparison to 360 *aṁśa*, and (2) assigning 100 *aṁśa*to to each *nakṣatra* or constellation could avoid many errors while taking fractions.

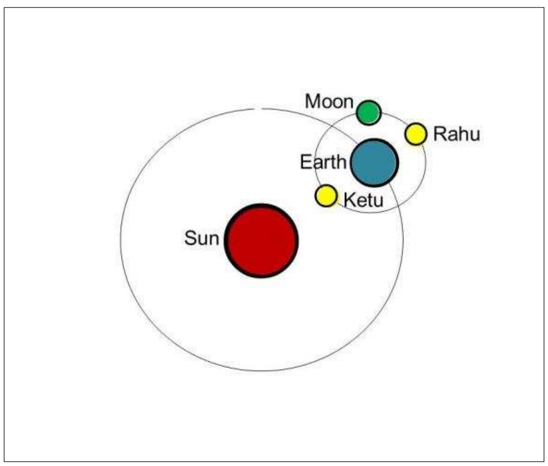

Figure 1: A schematic diagram (not to scale) showing the relative positions of the Sun, the Earth and the Moon for the calculations of the times of solar and lunar eclipses (diagram: Sudhira Panda).

4.2 Calculating Time According to the *Bhāsvatī*

In this section we want to show the simplified method introduced in the *Bhāsvatī* to calculate time from gnomonic shadows.

As an example: calculation of time on 15 June of this year (2019), when the shadow of the 12 unit gnomon becomes 15 units.
Answer: Here the equinoxial day is 23 March.
So the number of days elapsed = 8 days of March + 30 days of April +31 days of May +15 days of June = 84 days
or 30 days Aries + 30 days Taurus + 24 days Gemini = 84 days
Now to calculate *carārdhalitā*
for the month of Aries = 30+30/2 = 45
for the month of Taurus = 30+30/6 = 35
for the month of Gemini = 24/2 = 12
So *carārdhalitā* = 45 + 35 + 12 = 92 = *danda*1|32 *lita* on the day required
Dinārdha = 15 +1|32= 16|32 *daṇḍa*

Now, to calculate *madhya prabhā* (which is the mid-day Sun's rays)

Carārdhalitā 92 × 6 = 552/10 = 55|12
552 – 55|12 = (496|48)/10 = 49|41
On 15 June the Sun is in the northern hemisphere. So the above number should be kept as it is.
Now 49|41 – *akṣa* 44|43 = 4|58 → *madhya prabhā*
Here the gnomonic shadow or *iṣṭachāyā* = 15|0 *aṅgula* × 10 = 150 + 100 = 250
250 – *madhya prabhā* 4|58 = 245|02 = 245 × 60 +2 = 14702 → *śaṅku*
now*dinārdha* 16|32 = 16 × 60 + 32 = 992
992 × 100 = 99200
99200/14702 = *daṇḍa* 6|45 *litā*
Now we have to convert this to modern time.
daṇḍa 6|45 *litā* ~2 hours and 42 minutes
We know that in Indian astronomy the day starts at sunrise.
Dinārdha on 15 June is 16|32 ~6 hours and 22 minutes = $6^h\ 22^m$
Mid-day at 87° longitude is at $12^h - 14^m = 11^h\ 46^m$
Therefore, $11^h\ 46^m - 6^h\ 22^m = 5^h\ 24^m$, which is the time of sunrise.
$5^h\ 24^m + 2^h\ 42^m = 8^h\ 06^m$ is the required time when the shadow of 12 *aṅgula śaṅku* becomes 15 *aṅgula*.

4.2.1 A Physical Explanation to all the Terms and the Methods Adopted

To know time from the gnomonic shadow there are two terms that are involved in the calculation:

(1) *Madhyaprabha*, and
(2) *Dinārdhadaṇḍa*

Then, for the calculation *of Madyaprabha* and *Dinārdhadaṇḍa* we need to calculate *carārdha*, *nāḍi* and *nata*. *Nata* has two parts, *saumyanata* and *yamyanata*.

The first step of this method is to decide whether the Sun is in the northern or southern sky. If the Sun is in the north then *akṣa* has to be subtracted (otherwise it would have to be added). This is because when he wrote the *Bhāsvatī,* Śatānanda had made all his calculations with reference to Puri, Odisha, which is in the northern hemisphere. Therefore, when the Sun travels from the northern to the southern hemisphere it has to pass the equator, the zero equinoxial gnomonic shadow line. Hence, to consider the gnomonic shadow when the Sun is in southern hemisphere the term *akṣa* has to be added. According to the *Bhāsvatī*, the Sun lies in the northern hemisphere, from the vernal equinox to the autumnal equinox, for 187 days (the modern value is 186 days), while it is in the southern hemisphere, from the autumnal equinox to the vernal, for 178 days (the modern value is 179 days).

In the second step we have to calculate *carārdha* (spreading). As we know, the duration of the day and the night changes every day and is not completely uniform. Therefore to take care of the changes in a day, the duration *carārdha* has to be calculated. This is an empirical method and Śatānanda claims that the method is completely his own and that he did not copy from any previous texts. From *Madhyaprabhā* the midday gnomonic shadow for the day concerned can be derived. From the proportion of *Madyaprabhā* and *Iṣṭachāyā* the time can be calculated.

Dinārdhadanda can be calculated by adding *carārdhalitā* to, or subtracting it from, the *dinārdhadaṇḍa* on *Mahāviṣuvasaṅkrānti* (i.e. 15 *daṇḍa*, depending on whether Sun is in the northern or the southern hemisphere). Table 3 lists the midday gnomonic shadow on all 12 *saṅkrāntis,* along with modern data.

The length of the shadow of the gnomon should be recorded at the moment at which the time has been calculated. This is known as *iṣṭachāyā*.

iṣṭachāyā × 10 + 100 – *Madhya prabhā* = *Śaṅku* (2)

(Note that this *Śaṅku* is different from the gnomon itself.)

Keep *dinārdha* (half day duration) of that day. Convert *daṇḍa* and *litā* into *litā* by multiplying 60 with *daṇḍa* and then adding *litā*. Now multiply *litā* pind with 100 and then divide it by the value of *Śaṅku* in equation (2). The result is the *iṣṭachāyākāla* (time). This time is of two types, *Gatakāla*: from morning up to noon, and *Eṣvakāla*: from noon through to the evening.

To know *madhya prabhā* the *carārdhalitā* has to be calculated. Multiply 6 with *carārdhalitā*. Keep the result in two places. Subtract one tenth of it from the number in the second place. If the Sun is in the northern hemisphere then keep the number as it is, otherwise add one third of the number to it. Again divide the number by 10. If the Sun is in southern hemisphere then *akṣa* has to be added.

Śatānanda claimed in the *Bhāsvatī* that this method of calculation of *Carārdha* outlined there was entirely his own. According to him, if the Sun is in Aries (*Meṣa*), then the day count + the half of the day count is the *carārdhalitā*. If the Sun is in Tarus (*Vṛṣa*) then *Carārdha* will be the *carārdhalitā* of *Meṣa* + number of days elapsed from *Vṛṣa* + one sixth of number of days elapsed from *Vṛṣa*. Again, if the Sun is in Gemini (*Mithuna*), the half of the days elapsed from the month of Mithuna have to be added to the *carārdha* of the month *Vṛiṣa*. The result is the *carārdhalitā* for the month of Gemini (*Mithuna*). The *carārdhalitā* for the months of *Karkaṭa* to *Kanyā* will decrease in the similar manner, and on *Kanyā saṅkrānti* it will be zero. A similar calculation has to be followed if the Sun is in the southern hemisphere.

The half day duration, *dinārdha*, on *Mahāviṣuvasaṅkrānti* is 15 *daṇḍa*. Calculate the *carārdhalitā* for the day concerned, add the *carārdhalitā* to 15 if the Sun is in the northern hemisphere and subtract it if the Sun is in southern hemisphere. The result is the required *dinārdha* (half day duration) for the day concerned.

Since Śatānanda made all his calculations with respect to *ahargaṇa*, in order to make all of my calculations in same reference frame I adopted the data provided by NASA. The old data table by NASA is given below, where 21 March has been taken as *Mahāvisuva Saṅkrānti* or *Meṣa saṅkrānti*. In the *Bhāsvatī*, Śatānanda mentions that the Sun lies in the northern hemisphere for 187 days and in the southern hemisphere for 178 days, which is the same as in the NASA table.

Table 3: The mid-day gnomonic shadow on all 12 *Sankrānti.*

Saṅkrānti Number	Declination of the Sun (δ) in degrees	Right ascension of the Sun (λ) in degrees	Midday gnomonic shadow from the modern method	Midday gnomonic shadow from themethod in the *Bhāsvatī*	Difference and Error (%)
1	0.0	0.0	4.3676	4.45	0.0824 = 0.69%
2	11.5008	30.0	1.7933	1.9788	0.1855 = 1.55%
3	20.2017	60.0	0.04225	0.098	0.0557 = 0.46%
4	23.5	90.0	–0.7339	–0.658	0.0759 = 0.63%
5	20.2017	120.0	–0.04225	–0.037	0.0795 = 0.66%
6	11.5003	150.0	1.7933	1.739	–0.543 = 0.45%
7	0.0	180.0	4.3676	4.45	0.082 = 0.69%
8	–11.5004	210.0	7.3537	7.496	0.1423 = 1.19%
9	–20.2017	240.0	10.1414	10.232	0.0906 = 0.75%
10	–23.5	270.0	11.3875	11.24	–0.1475 = 1.23%
11	–20.2017	300.0	10.1414	10.148	0.0066 = 0.05%
12	–11.5008	330.0	7.3537	7.595	0.2413 = 2.025%

5 CONCLUDING REMARKS

In this paper, the contribution of Śatānanda to the world of mathematics and astronomy has been discussed. Some of the *ślokas* from his text *Bhāsvatī* has been translated to explain his achievements. It was necessary in order to prepare an accurate almanac for Hindu society, and mostly for the benefit of the Jagannātha temple at Puruṣottamadhāma Purī. For this purpose he applied the observational data of Varāhamihira and took CE 450 as the year when the text of the *Pañcasiddhāntikā* of Varāhamihira was written, as zero *ayanāṁśa*' year. Śatānanda started *Śāstrābda* from the year he dedicated the *Bhāsvatī* to society, i.e. CE 7 April 1099 (Mishra 1985). Correspondingly, it was the *Pournim*a (Full Moon day) of the first lunar month *Caitra* of the *gata-kali* (elapsed *kali* era) year 4200. All calculations in the *Bhāsvatī* were in *Śastrābda*, and he had given rules to convert *Śāstrābda* to *Śakābda* and *vice versa*. Śatānanda has taken the latitude and longitude of Puri in Odisha as his reference point. Maybe it was easy for him to recheck his methods from observations made at his native place.

The most interesting thing found in the *Bhāsvatī* is that Śatānanda could calculate the position and rate of motion of heavenly bodies quite accurately without using trigonometric functions. Though some ancient astronomers rejected the methodology by saying that was an approximate method, it is interesting to see that this 'approximate method' could provide exact solutions when predetermining eclipses. Use of *Śatāṁśa* (a centesimal system) in the procedure and making a back transform was quite a modern idea that was adopted by Śatānanda. A strong claim exists that the conversion of the sexagecimal system to the centesimal system was the first step that led mathematicians towards the introduction of the decimal system in mathematical calculations (Vaidya, 1981: 110–112). In this context, it is necessary to study the physical and mathematical interpretation of all 128 *ślokas* in the *Bhāsvatī*.

A detail study is now in progress to establish the relationship between the method outlined in the *Bhāsvatī* and the modern European method of predetermining an eclipse.

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8 APPENDIX A: THE METHOD OF CALCULATING THE SIDEREAL PERIOD OF MOON

Step 1. Multiply 90 by *ahargaṇa* and add *Candra Dhruva* with it. Divide the result by 2457.

Step 2. Multiply 100 by *ahargaṇa* and add *Kendra dhruva* to it. Divide the result by 2756.

Step 3. Divide *ahargaṇa* by 120 and add the remainder of Step 1. The *carārdha* of the respective month has to be subtracted from the result. The *carārdha* for each month is given in Table 4.

Step 4. Divide *ahargaṇa* by 50 and add the remainder of Step 2. Then divide the result by 100.

Step 5. From the quotient the corresponding *Khaṇḍa* and *Anukhaṇḍa* (*khaṇḍa* +1) have to taken from Table 5 below. Subtract *Khaṇḍa* from *Anukhaṇḍa*, and the result is *chandra bhoga*. The remainder from Step 4 has to be multiplied by *chandra bhoga*. Divide the result by 100. The result has to be added to *Khaṇḍa* and the result of Step 3. The result is *candra sphuṭa*.

In the similar manner *candra sphuṭa* for the next day (*ahargaṇa*) has to be calculated. The positional difference of the day is called *candra bhukti* (the Moon's diurnal motion). This motion is not uniform. Therefore for the sidereal calculation I kept on increasing the *ahargaṇa* until the Moon comes to the same position (*candra sphuṭa*).

Table 4: The Carardha value that has to be subtracted in different months

Name of Sidereal Month	*Carārdha*	Name of Sidereal Month
Aries	0	Pisces
Taurus	1	Aquarius
Gemini	2	Capricorn
Cancer	2	Sagittarius
Leo	1	Scorpio
Virgo	0	Libra

Table 5: *Candra Khaṇḍa*-difference (*antara*) – *Bhuktibodhaka Chakra*

0	1	2	3	4	5	6	7	8	Number
0	0	1	3	6	10	16	24	35	*Khaṇḍa*
0	1	2	3	4	6	8	11	11	Difference
9	10	11	12	13	14	15	15	17	Number
46	60	75	91	108	1	143	159	175	*Khaṇḍa*
14	15	16	17	18	17	16	16	15	Difference
18	19	20	21	22	23	24	25	26	Number
190	202	213	222	230	235	239	241	242	*Khaṇḍa*
12	11	9	8	5	4	2	1	1	Difference

27	28	Number
243	243	*Khaṇḍa*
0	0	Difference

9 APPENDIX 2: THE METHOD OF CALCULATING THE MID-DAY GNOMONIC SHADOW IN DIFFERENT *SAṄKRĀNTIS*

The NASA table for different *Saṅkrāntis*:

[redacted]

According to the *Bhāsvatī*, the *palaprabhā* (equinoxial mid-day gnomonic shadow) is 4|27 = 4.45
This is little higher than that of modern data (i.e. 4.37 + 0.08)

1. On 21 March the Sun lies on the equator. So we take the Sun's position at 0°. Aries.
So the gnomonic shadow will be 4.45.
2. On 21 April, Taurus = 30°= *ahargaṇa* = 31 = 30 +1
carārdhalitā = 45 + 1 + 1/6 = 46.17
46.17 × 6 = 277.02 – 27.70 = 249.32/10 = 24.932
madhya prabhā = 44.72 – 24.932 = 19.788
iṣṭachāyā = 19.788/10 = 1.9788
3. On 22 May, Gemini: 60°= *ahargaṇa* 62 = 30 + 30 + 2
carārdhalitā = 45 + 35 + 1 = 81
81 × 6 = 486 – 486/10 = 437.4/10 = 43.74
madhya prabhā = 44.72 – 43.74 = 0.98
iṣṭachāyā = 0.98/10 = 0.098
4. On 22 June, Cancer: 90°= *ahargaṇa* 93 = 30 + 30 + 33
carārdhalitā = 45 + 35 + 33/2 = 96.5
96.5 × 6 = 579 – 57.9 = 521.1/10 = 52.11
madhya prabhā = 44.72 – 52.11 = –7.39
iṣṭachāyā = –7.39/10 = –0.739
5. On 23 July, Leo: 120°= *ahargaṇa* 124
(In this case there is little change in procedure. It has been mentioned that the Sun lies 187 days in the Northern Hemisphere and 178 days in the Southern Hemisphere. So when *ahargaṇa* exceeds half of the days in a hemisphere then we have to take the smaller part for the *carārdhalitā* calculation. i.e. 187 – 124 = 63. So we have to calculate the *carārdhalitā* of 63 *ahargaṇa*.)
63 = 30 + 30 + 3
carārdhalitā = 45 + 35 + 3/2 = 81.5
81.5 × 6 = 501 – 50.1 = 450.9/10 = 45.09
Madhya prabhā = 44.72 – 45.09 = –0.37
iṣṭachāyā = –0.37/10 = –0.037

6. On 22 August, Virgo: 150° = *ahargaṇa* 154 = 187 – 154 = 33 = 30 + 3
carārdhalitā = 45 + 3 + 3/6 = 48.5
48.5 × 6 = 291 – 29.1 = 261.9/10 = 26.19
madhya prabhā = 44.72 – 26.19 = 18.53
istachaya = 18.53/10 = 1.853
7. On 24 September, Libra: 180° = *ahargaṇa* 187
Shadow length = 4.45
8. On 22 October, Scorpio: 210° = *ahargaṇa* 215
215 – 187 = (Southern Hemisphere) = 28
carārdhalitā = 28 + 14 = 42
(there is little change in procedure for the Southern Hemisphere)
42 × 6 = 252 – 25.2 = 226.8 + 226.8/3 = 302.4/10 = 30.24
madhya prabhā = 44.72 + 30.24 = 74.96
iṣṭachāyā = 74.96/10 = 7.496
9. On 23 November, Sagittarius: 240° = *ahargaṇa* 247
247 – 187 = 60
carārdhalitā = 45 + 35 = 80
80 × 6 = 480 – 48 = 432 + 432/3 = 576/10 = 57.6
madhya prabhā = 44.72 + 57.6 = 102.32
iṣṭachāyā = 102.32/10 = 10.232
10. On 23 December, Capricorn: 270° = *ahargaṇa* 277
277 – 187 = 90
Southern Hemisphere 178 – 90 = 88
We have to calculate *carārdhalitā* of the smaller part.
So *carārdhalitā of 88* = 45 +35 +14 = 94
94 × 6 = 564 – 56.4 = 507.6 + 507.6/3 = 676.8/10 = 67.68
madhya prabhā = 44.72 + 67.68 = 112.4
iṣṭachāyā = 112.4/10 = 11.24
11. On 21 January, Aquarius: 300° = *ahargaṇa* 306
306 – 187 = 119
178 – 119 = 59
59 = 45 + 29 + 29/6 = 78.83
78.83 × 6 = 473 – 47.3 = 425.7 + 425.7/3 = 567.6/10 = 56.76
madhya prabhā = 44.72 + 56.76 = 101.48
iṣṭachāyā = 101.48/10 = 10.148
12. On 20 February, Pisces: 330° = *ahargaṇa* 336
336 – 187 = 149
178 – 149 = 29
29 + 29/2 = 43.5
43.5 × 6 = 261 – 26.1 = 234.9 + 234.9/3 = 312.3/10 = 31.23
madhya prabhā = 44.72 + 31.23 = 75.95
iṣṭachāyā = 75.95/10 = 7.595

Dr Sudhira Panda is a former Lecturer in the Department of Mathematics at Ravenshaw University, Cuttack, India. She did her PhD at the University of Hong Kong. She has been doing research on optoelectronic device modeling, funded by the Department of Science and Technology and the Council of Scientific and Industrial Research, India. During the last five years she has been researching aspects of ancient Indian astronomy.