Variability Constraints on Blazar Magnetic Fields

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Abstract: Synchrotron self-Compton (SSC), external Compton, and hadronic models of blazar emission all invoke particle acceleration at relativistic shocks as the dissipation mechanism seeding their non-thermal X-ray and gamma-ray emission. Studies of diffusive acceleration at such relativistic shocks are more sparse than those pertaining to their non-relativistic counterparts. This paper presents acceleration time results from the theory of relativistic shock acceleration that are pertinent to AGN observations. This temporal information interfaces critically with the observed rapid variability of blazars. Very recent theoretical results are presented, where it is determined that acceleration times can never become arbitrarily short in relativistic shocks, but are dominated by diffusion in the downstream region and couple to the particle's gyroperiod. This fundamental acceleration bound links to the variability timescale to generate a firm lower bound to the environmental magnetic field of blazars such as Mrk 421. Consistency of such a bound with SSC spectral models and flare decay times is discussed.

Keywords: acceleration of particles — shock waves — relativity — radiation mechanisms: non-thermal — galaxies: active

1 Introduction

The history of our understanding of blazars has largely been inseparable from their variability, the key distinguishing characteristic of their ‘parents’, BL Lac objects. The blazar class became more distinctive following the discovery by the Compton Gamma-Ray Observatory that they were extremely bright and rapidly variable gamma-ray emitters. Since the X-ray and gamma-ray bands generally dominate the blazar energetics during flares, the properties of generation of ultrarelativistic particles in blazar environs on short timescales become salient. Such particles, presumably electrons, are commonly postulated to be responsible for inverse Compton emission in the gamma-ray band, and synchrotron emission in the radio to X-ray wavelengths. The critical division in the community is over whether the seed photons for the inverse Compton signal are cospatially generated synchrotron emission (e.g. Maraschi, Ghisellini, & Celotti 1992), or originate from distinct regions such as an accretion disk or from scattering clouds (e.g. Dermer, Schlickeiser, & Mastichiadis 1992; Sikora, Begelman, & Rees 1994). Current thought favours the former case significantly for the specific cases of Mrk 421 and Mrk 501. However note that blazars such as 3C 279 possess manifestly different spectral character (Hartman et al. 2001).

Motivations for variability studies include probing the properties of acceleration and radiative cooling. Blazar variability probably reflects intrinsic variations in the particle population and ambient magnetic field. Current SSC blazar models suggest (e.g. Krawczynski et al. 2001) that flaring accompanies increases in electron Lorentz factors, and hence the rise times must couple to acceleration rates. Furthermore, cooling timescales cannot be inferior to flare durations, unless they are dominated by acceleration, thereby imposing cooling bounds to fields and particle energies. Invariably, the most telling constraints come from the highest energy particles, and therefore correlate with X-ray and TeV gamma-ray data. Such radiation must be generated by super-TeV particles, which probably arise from diffusive acceleration in shocks. The shocks are inferred to be relativistic in nature, based upon much evidence for superluminal motion, and the ‘relativistic beaming’ constraints obtained (e.g. Ghisellini et al. 1993) from the absence of $\gamma \rightarrow e^+ e^-$ attenuation of gamma rays in blazars. Current observed variabilities in the X-ray and TeV bands are typically in the 5–60 min range (e.g. for Mrk 421, see Maraschi et al. 1999, Krawczynski et al. 2001; for Mrk 501, see Catanese & Sambruna 2000), and therefore are quite constraining: as will become apparent, they make the case for an external Compton model perhaps more difficult for these sources.

2 Acceleration Theory for Relativistic Shocks

The study of diffusive (Fermi) particle acceleration at relativistic shocks has historically been sparser than for their non-relativistic counterparts for a number of reasons. The only cosmic locales where particle acceleration is measured in situ are necessarily heliospheric, and therefore intrinsically non-relativistic. With the possible exception of recently discovered galactic superluminal sources, generally galactic shock environs involve non-relativistic flows, motivating the historical emphasis. The connection of cosmic rays to relativistic shock environs is mostly made in extragalactic contexts, specifically active galaxies and gamma-ray bursts, and then usually for those particles of the highest energy, i.e. those above $10^{18}$ eV.

From a theoretical perspective, favouring non-relativistic shocks in studies of diffusive acceleration is
natural, since they generate isotropy in their relativistic particles; this is not true for relativistic shocks where the fundamental limiting nature of $c$ comes into play. Such isotropy dramatically simplifies analytic treatments, and affords several compact results, such as the determination of acceleration times (e.g. see Forman, Jokipii, & Owens 1974). After occasional early works, more detailed analyses of Fermi acceleration at relativistic shocks started appearing during the late 1980s. Two principal works, for so-called plane-parallel shocks, where the field is normal to the shock plane, were the semi-analytic convection–diffusion equation approach of Kirk & Schneider (1987), and the Monte Carlo simulation technique of Ellison, Jones, & Reynolds (1990, hereafter EJR90). While complementary, the two methods were shown to agree in their spectral index determination when common parameter choices were invoked. Moreover, recent extensions of these two approaches, namely Kirk et al. (2000), and M. G. Baring & F. C. Jones (in preparation, hereafter BJ02), demonstrate agreement (BJ02) with computation of angular distributions, a critical test on the accuracy of the Monte Carlo simulation.

A principal conclusion of the earlier works was that, for a given ratio of upstream to downstream fluid speeds in the shock frame, increasing the shock speed to relativistic values flattened the particle distribution. This persisted when oblique shocks (i.e. those where the field lines are oblique to the shock normal) were examined (Kirk & Heavens 1989; Bednarz & Ostrowski 1998). The physical origin of this property is that the increase in the energy gain $\Delta E$ per shock crossing cycle with upstream fluid speed $\beta_c c$ outweighs the accompanying increase in particle loss rates per cycle. The energy gains, loss rates, and consequently spectral indices depend on the type of scattering assumed (e.g. Baring 1999). Small-angle scattering, corresponding to angles less than $1/\Gamma_1$ (e.g. pitch angle diffusion), yields inefficient convection against the fast upstream flow, so that spectra are steeper than for the case of large angle (i.e. $\gtrsim 1/\Gamma_1$) scattering, where energy gains are a sizeable fraction of $\Gamma_1^2$. As with much previous work on relativistic shocks, our focus hereafter will be on parallel shocks, which are technically simpler than oblique and quasi-perpendicular shocks. This specific choice is not unrepresentative of oblique shocks in AGN provided the field turbulence is strong, so that scattering is near the Bohm diffusion limit.

One of the profound advantages of simulations in general, and Monte Carlo ones in particular, is that they can readily compute the efficiency with which particles are accelerated from thermal energies (e.g. Ellison, Baring, & Jones 1995), and furthermore calculate cumulative acceleration times. Semi-analytic diffusion–convection equation techniques obtain spectral indices in shocks of any speed, but must parameterise injection from the thermal particle pool. At present, they have not been used to compute acceleration times in relativistic shocks, which was first done with a Monte Carlo method by EJR90. This situation arises because of the intrinsic anisotropy of the distributions when $\Gamma_1 \gg 1$, the consequence of the inability of particles to stream against the flow. EJR90 found that for large angle scattering, the acceleration time for a $\Gamma_1 \lesssim 5$ shock was only marginally shorter than that expected from classical non-relativistic shock theory. While confirming these early results, BJ02 have recently computed acceleration times in the limit of pitch angle diffusion (see also simulation results of Bednarz 2000; Achterberg et al. 2001). They found that extrapolation of simulations into the relativistic regime revealed a hard lower bound on the total acceleration time $\tau_{acc}$ as measured in the shock rest frame.

The time $\tau_{acc}$ monotonically decreases (for ultrarelativistic particles) to this limit as $\Gamma_1$ increases to infinity, yet proximity is achieved for $\Gamma_1 \cong 10$. If $v_g$ represents the energy-dependent gyrofrequency of an ultrarelativistic electron or ion, then the velocity dependence of the acceleration times in plane-parallel shocks, as determined by BJ02, can be approximated (to around 1–3% accuracy) by the empirical fit

$$\tau_{acc} \approx \left( \frac{0.25 - 0.18}{\Gamma_1 \beta_1} + \frac{1}{\Gamma_1^2 \beta_1^2} + \frac{0.22}{1 + \Gamma_1 \beta_1} \right) \times \tau_{NR}(\beta_1 = 1), \quad \tau_{NR}(\beta_1 = 1) = \frac{f}{v_g}. \quad (1)$$

Here $\tau_{NR}(\beta_1 = 1)$ is the extrapolation of the well-known acceleration time formula for non-relativistic shocks to flow speeds $c$. The times are for a relativistic equation of state, i.e. a velocity compression ratio of $\beta_1/\beta_2 = 3$, and the coefficient $f$ describes details of the differences in diffusion between the upstream and downstream regions, and is of the order of unity and independent of $\Gamma_1$.

The bound arises due to the insensitivity of the downstream flow speed and diffusion in the downstream region to the upstream $\Gamma_1$. The limiting angular distribution at the shock was found by Kirk et al. (2000; confirmed numerically by BJ02), and is essentially controlled by downstream diffusion independent of beaming upstream of the shock. Such downstream diffusion yields the dominant contribution to $\tau_{acc}$, with upstream particles requiring only small deflections from the shock normal in order to return downstream, thereby automatically implying the hard bound as $\Gamma_1 \to \infty$. Effectively, particles can never be accelerated at rates much faster than their gyrofrequency. The limit generates a comparable limit in the upstream fluid frame, which is often the observer’s reference perspective. This consequence follows (BJ02) from the connection between Lorentz transformations of times and energies, with the proper time of the particle being an invariant. This bound is the principal shock acceleration result that is now exploited in the discussion on blazars.

### 3 Implications for Blazars

The coupling of theoretical acceleration and cooling times to observed blazar temporal variability timescales provides interesting and useful interpretative diagnostics.
The above exposition indicates that the mildly relativistic shocks expected in blazar jets possess a well-confined range of acceleration times, perhaps within a factor of a few, for a given upstream magnetic field $B_1$ in the shock frame. This field is related to the environmental field $B_e$ in the observer’s frame of reference by a Lorentz transformation, and accordingly lies somewhere in the range $B_e \approx B_1 \leq \Gamma_1 B_1$, depending on the orientation of the field vectors to the observer’s line of sight. This field amplification is at most of the order of a few or 10 for blazars, and therefore is modest when compared with gamma-ray burst scenarios. The numerical value of $\tau_{\text{acc}}$ in the shock frame corresponding to $\tau_{\text{NR}}(\beta_1 = 1)$ in equation (1) can be adapted from equation (13) of Baring et al. (1999):

$$\tau_{\text{acc}} \approx \frac{0.1 \, E_{\text{TeV}}}{\beta_1^2 \, B_{\text{Gauss}}} \, \text{sec} \lesssim \tau_{\text{var, rise}},$$

where the subscripts ‘TeV’ and ‘Gauss’ denote the units that scale the particle energy $E$ and upstream field $B_1$, respectively. As discussed above, this acceleration timescale can be interpreted in either the shock rest frame or the observer’s (upstream) frame, with values transforming by factors of at most a few. It must be smaller than the rise time $\tau_{\text{var, rise}}$ measured in the observer’s frame for a blazar flare, implying a lower bound to the environmental field $B_e$ and an upper bound on the particle energy. Such a constraint is plotted in Figure 1 for the fiducial variability timescale of $\Delta t \sim 300$ sec, which is probably an upper limit to the intrinsic $\Delta t$.

It is routine to also place a cooling time constraint on the same plot, to narrow the permitted phase space. The synchrotron cooling timescale, $\tau_{\text{syn}} \approx \frac{300}{B_{\text{Gauss}}^2 \, E_{\text{TeV}}} \, \text{sec} \gtrsim \tau_{\text{var, drop}}$, is germane to the discussion of the TeV blazars Mrk 421 and Mrk 501, with the inverse Compton cooling time being somewhat longer for the highest energy ($\gamma \sim 10^6$) electrons, regardless of whether it originates in a synchrotron self-Compton (SSC; see Li & Kusunose 2000 for a discussion) or external Compton (EXT) scenario. This contrasts the case of 3C 279, for which the gamma rays dominate the power during flares (Hartman et al. 2001), and Compton cooling is expected to be extremely efficient. For Mrk 421 and Mrk 501, $\tau_{\text{syn}}$ must exceed the timescale $\tau_{\text{var, drop}}$ ($\sim 300$ sec inferred from X-ray data; the intraday radio variability that has been widely discussed is probably due in part to interstellar scintillation) for the decline during, or at the end of, a blazar flare; otherwise once acceleration shuts down, the flux would drop off more precipitously than observed. Hence we arrive at an additional upper limit to both $B_1$ and $E$ that is depicted in Figure 1.

The permitted phase space is a triangle to the left of the plot, extending downwards from a particle energy of around 200 TeV; this represents the maximum expected energy (electron or ion) in blazar flares if a shock acceleration scenario is applicable. Clearly shortening the variability timescale tightens the acceleration constraint, but relaxes the synchrotron one, generally moving the triangle to the upper left of the plot.

This phase space diagram can quickly be compared with energies and fields expected from spectral fitting of the flares. For SSC models these are of the order of $B_1 \sim 0.1$ Gauss and $E_{\text{max}} \sim 0.3$ TeV (e.g. see Takahashi et al. 2000; Krawczynski et al. 2001), and are easily accommodated by the present variability constraints. External Compton models can have somewhat different field particle energies: for example Dermer, Sturmer, & Schlickeiser (1997) obtain $B_1 \sim 5$ Gauss and $E_{\text{max}} \sim 0.2$ TeV in fitting Mrk 421. Accordingly acceleration constraints are not definitively limiting for either SSC or EXT models, however synchrotron cooling provides marginal problems for the EXT model.

The principal conclusion of this paper is that the absence of relativistic effects in the determination of acceleration times leads to a more or less precise positioning of bounds on the magnetic field/energy diagram. The combination of spectral and temporal constraints for Mrk 421 and Mrk 501 cannot yet discriminate between SSC models and external Compton ones, though the boundaries of viability are interestingly placed; any future detection of shorter rise and decay times may force model revisions.

References
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