High azimuthal number pulsation modes in fast rotating δ Scuti stars: the case of HD 101158 ≡ V837 Cen*

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Received 18 March 1996 / Accepted 13 September 1996

Abstract. The frequency analysis of the line profile variations of the fast rotating (v sin i = 132 km s\(^{-1}\)) δ Scuti star HD 101158, observed for three consecutive nights, shows the presence of two high azimuthal number non radial pulsation modes. The star is probably seen almost equator-on and both modes (\(\nu_1 = 12.9\) c/d and \(\nu_2 = 18.5\) c/d) are prograde with \(m = -10\) and \(m = -14\) or \(-15\) respectively; their frequencies are different with respect to the three frequencies identified in photometric data (Poretti 1991), which probably owe due to low \(\ell\) modes. Indications of the presence of these photometric modes have been found from the frequency analysis of the first two line moments. The line profile variations also show the possible presence of further modes with frequencies of 16.2, 20.3 and 21.1 c/d and small amplitudes.

Key words: data analysis – spectroscopy – stars: δ Scuti – stars: individual: HD 101158 – stars: oscillation of

1. Introduction

The light variations of the δ Scuti star HD 101158 have been carefully studied by Poretti (1991) who detected three pulsation modes with frequencies of 9.27, 10.83 and 12.16 c/d. Since the star seemed to have a quite simple pulsational spectrum and it is quite bright it was considered a good target for a campaign of simultaneous photometric and spectroscopic observations in order to typify the pulsation modes. To this end observing time was allotted at ESO in April 1994 at the CAT and at the 0.5 m telescope. Unfortunately, due to technical reasons, the photometric run had to be cancelled, and therefore only spectroscopic measurements were performed.

2. Observations and data reductions

The observations were made at La Silla Observatory during three consecutive nights (April 1–3, 1994) with the Coudé Echelle Spectrograph (CES) attached to the Coudé Auxiliary Telescope (CAT) in the Short Camera. The spectral resolution was 60 000 and the useful spectral region was from 4490 to 4526 Å (central wavelength 4508.0 Å). The spectrograms were acquired through the ESO CCD #9, and have a linear dispersion of 0.036 Å/pix. A total of 74 spectrograms with exposures of 15 min each were obtained covering a total observing time of 24 hours. The average S/N ratio of the spectrograms at the continuum level is about 200. The stellar spectra were extracted from the CCD images and calibrated into wavelengths using the MIDAS package. The line profiles show that the star is a quite fast rotator, so that, due to the blending of adjacent features, only two lines are suitable for the subsequent analysis: Ti ii 4501.28 Å and Fe ii 4508.29 Å. The spectrograms were therefore normalized to the stellar continuum in the range 4494–4518 Å by means of a 2nd degree polynomial obtained through a least-squares fitting of a few spectral continuum windows selected from the average spectrum. The average spectrum is reported in Fig. 1. The lines have the typical profile caused by rotational broadening, even if they are not perfectly symmetrical and the 4501 Å line shows some sort of refilling at its bottom. The Fe ii+Cr i 4515.35 Å line, because of the inclusion of some weak lines on its wings, has a FWHM larger than those of the two other lines and cannot be used to study the propagation of perturbations along the profile.

From these profiles we have estimated the projected rotational velocity by fitting them with a rotationally broadened gaussian profile, whose width was determined by the physical parameters derived from \(uvbyβ\) photometry (Hauck & Mermilliod 1990) and by the instrumental profile. From the calibration of Moon & Doworetsky (1985) we get \(T_{\text{eff}} = 7600\) K and \(\log g = 3.64\) and from that of Crawford (1979) \(M_V = 1.12\), and consequently \(R \simeq 3R_⊙\). So from the line profile fitting we estimated \(v_c \sin i = 132 \pm 3\) km s\(^{-1}\).

2.1. Frequency analysis of line profiles

From the profile of each line we can obtain 132 time-series corresponding to the temporal behaviour during the three observing nights of the individual wavelength pixels. The coincidence of
this number with the projected rotational velocity depends on the fact that the dispersion of the wavelength resampled spectrograms is about 2 km s\(^{-1}\) pixel\(^{-1}\). Each time series can then be independently frequency analysed, this approach is widely used for the study of line profile variations (see for example Reid & Aerts 1993, where a full reference list is given). The adopted frequency analysis technique is the least-squares multiple sinusoid fit defined by Vaniček (1971) and successfully used by us in the study of the photometric variability of several \(\delta\) Scuti stars (see for example Mantegazza et al. 1996 and for a list of many other recent papers Mantegazza & Poretti 1995).

It consists of the simultaneous least-squares fit of \(n + 1\) sinusoids where \(n\) represents the previously identified terms (known constituents, hereinafter k.c.) and the \(n + 1\) term is the sinusoid corresponding to the trial frequency. This technique is particularly suited for the study of multiperiodic light curves because it does not require any prewhitening of the data since the amplitudes and phases of the frequency terms previously identified, when searching for a new frequency term, are recalculated for each new trial frequency. Only the frequencies of the k.c. are kept constant, but their values are refined by a non-linear least squares fit after each new periodic constituent has been detected. The ordinates of the spectra show the reduction factor

\[
RF = 1 - \frac{\sigma^2_{\text{res}}}{\sigma^2_{\text{in}}}
\]

where \(\sigma_{\text{in}}\) is the residual variance around the computed curve before considering the new trial frequency and \(\sigma_{\text{in}}\) is the residual variance after considering it.

The power spectra obtained are shown Fig. 2 where the abscissa report the temporal frequencies and the ordinates the wavelengths corresponding to the line profiles (Ti\(\equiv\) 4402 Å on the left, Fe\(\equiv\) 4508 Å on the right). Upper panels show the spectra without known constituents, which clearly indicates the presence of two perturbations with frequencies of 12.92 and 18.50 c/d. Each perturbation covers most part of the line profiles and is characterised by its central peak flanked by two quite strong sidelobes at \(\pm 1\) c/d due to the fact that the observations are distributed over three successive nights.

Fig. 3, upper panel, shows that the power spectra averaged on all the wavelengths confirm the interpretation given above. The central panels of Figs. 2 and 3 show the power spectra introducing \(\nu = 12.92\) c/d as a known constituent: the other remaining perturbation emerges as the dominant feature. The bottom panels of Figs. 2 and 3 show the power spectra with \(\nu_1 = 12.92\) c/d and \(\nu_2 = 18.50\) c/d as known constituents: most of the signal has disappeared; there are however some indications of the presence of other weak terms with frequencies of about 16.2, 20.3 and 21.1 c/d. The noise in Fig. 2 seemingly increases from top to bottom panels because each panel has been normalized with respect to its maximum value.

Figs. 4 and 5 show the behaviours along the lines profiles of the amplitudes (left panels) and the phases (right panels) of the two unambiguously detected pulsation terms (i.e. 12.9 and 18.5 c/d) respectively. The two line profiles give concordant information for both terms. In particular the angular coefficient of the phase lines shows that both terms are prograde waves and the number of these lines indicates that the azimuthal number \(|m|\) should be quite high (\(> 8\)) for both terms.

Because of the projection effects at the approaching and receding limbs, the number of phase lines supplies only a lower limit to the correct \(|m|\) value. A more accurate estimate is given by the shift in velocity between the two central phase lines measured at the center of the line profiles (\(\Delta v\)). The separation in longitude between the two equiphasic points \(\Delta \phi\), assuming \(i \simeq 90^\circ\), is given by:

\[
\Delta \phi = 2 \arcsin \left( \frac{\Delta v}{2v_{\text{c}}^2} \right)
\]

and thus \(|m| = 360^\circ / \Delta \phi\). The two spectral lines give concordant results, we obtain \(|m| = 11.1 \pm 0.6\) and \(|m| = 14.5 \pm 0.6\) for \(\nu_1 = 12.9\) c/d and \(\nu_2 = 18.5\) c/d respectively. Another approach to evaluate \(|m|\) consists in comparing the observed curves with those derived from synthesized line profiles. This has been made by means of the LNPROM code, kindly supplied by L.A. Balona. Synthesized profiles with the time distribution of the observed spectrograms have been generated for a star with physical characteristics similar to those of HD 101158 and for different \(m = -\ell\) values, since usually, when high azimuthal numbers are discovered, it is assumed that we are in presence of sectorial \((\ell = |m|)\) modes (see for example Kennelly 1991, Kennelly et al. 1992a, b, Korzennik et al. 1995). However a recent paper by Schrijvers et al. (1996), has pointed out that also tesselar modes with small \(\ell - |m|\) value can give considerable moving patterns when seen at a moderate inclination (e.g. 50°); this possibility will be discussed below.

The shapes and spacings of phase curves depend also on the inclination of the rotational axis. In the case of HD 101158 we expect from the high projected rotational velocity that the star

\[
\Delta \phi = 2 \arcsin \left( \frac{\Delta v}{2v_{\text{c}}^2} \right)
\]
is seen almost equator-on. In this case the sensitivity of phase curves to the inclination is quite small. Simulations show that this effect is barely discernible for the 18.5 c/d term, while the 12.9 c/d term gives a slight preference for $i \simeq 75^\circ$ with respect to $90^\circ$.

Fig. 6 shows the observed phase curves of the two lines (crosses $\lambda 4501$, triangles $\lambda 4508$) for the two periods ($\nu = 18.5$ c/d upper panel, $\nu = 12.9$ c/d lower panel) with the computed lines for the models which best fit the data superimposed. The lines have been computed for $i = 75^\circ$ but it makes little difference (particularly for $\nu = 18.5$ c/d) if we use $i = 90^\circ$. For $\nu = 12.9$ c/d the best fit is for $m = -10$ (solid line; dashed line $m = -11$), while for $\nu = 18.5$ c/d it is difficult to decide between $m = -14$ (solid line) and $m = -15$ (dashed line). These values should be considered the best estimates, because referred to the information extracted from the whole line profile.

In principle it would be possible to get an estimation of the complete stellar pulsation characteristics by trying to fit the observed line profiles, but since the models rely upon too many free parameters (at least for this star) the attempt is without hope. So one should be content to get satisfactorily fits of the phase curves which allow to get estimates of the azimuthal numbers of the two spectroscopic dominant modes and their respective phases. Moreover for an assumed set of $\ell$, $m$, $i$ parameters we can estimate from the relative amplitudes of the two modes with respect to the central line depths the amplitudes of the pulsation velocities. In our case with $i = 75^\circ$, $\ell = -m = 10$ and 14 for $\nu_1 = 12.9$ and $\nu_2 = 18.5$ we get $v_p \simeq 1.7$ and $\simeq 2.0$ km s$^{-1}$ respectively. As an example Fig. 7 shows the observed profiles of 4502 Å Ti II line for the three observing nights with superimposed this solution. A few experiments have shown that the horizontal velocities should be small with respect to the vertical ones otherwise the relative amplitudes of the moving patterns would be larger in the line wings with respect to the centers. This is what we expect in $\delta$ Scuti stars which usually are pulsating with $p$ modes. Therefore we have assumed that the horizontal
velocities are in phase with the vertical ones and with amplitudes given by $v_h = 0.08 v_p$ (given their smallness it is easy to verify that more accurate values are unimportant). Finally we have neglected the effects of temperature variations that could be treated as a pseudovelocity field (Balona 1987).

As we can see from Fig. 7 the fit of the observed profiles is reasonable, moreover it has to be remembered that the integration time of 15 min is as much as 20% of the period corresponding to $\nu_2$, 13% of the one corresponding to $\nu_1$ and 6% of their beat period. This could explain the asymmetric average profile and also tells us not to expect perfect fits from the LNPROF code that assumes instantaneous observations.

Following the above quoted results by Schrijvers et al. (1996), a few experiments have been performed to make fits with $\ell = 11$, $m = -10$, for $12.9 \text{ c/d}$, $\ell = 15$, $m = -14$, for $18.5 \text{ c/d}$ and $i = 50–70^{\circ}$. Comparable results can be obtained in the line centers, but worse in the wings. This does not imply that with an adequate adjustment of the many free parameters tesseral modes, when seen at moderate inclinations, could not supply comparable or even better fits. As said above, the fit shown is only an example, it is not to be considered the best one since there are too many free parameters to adjust. The quite high projected rotational velocity tends however to favour the hypothesis that the star is seen almost equator-on.

### 2.2. Photometric modes

Given the high $|m|$ values the two spectroscopically detected modes cannot generate appreciable light variations, so it is not surprising that their frequencies do not correspond to any of those derived by Poretti (1991) from the light curve analysis.

In order to check for the signature in the line profile variations of the photometric modes the first moments of the line profiles have been computed, since it is expected that these quantities are essentially sensitive to low degree modes, i.e. to those responsible for light variations. They have been evaluated for the two lines following the prescriptions given by Balona (1986a), and then they have been averaged. The resulting time series are
quite noisy and the noise level increases rapidly as the moment order increases, so that significant results from the frequency analysis can be found only for the first two moments. The time series of the two moments are shown in Fig. 8, while Fig. 9 shows their frequency analysis. We see that:

\(\langle v \rangle\) (radial velocity; top and middle panel) gives a dominant peak at 9.81 c/d and in the subsequent spectrum, after a low frequency peak (1.05 c/d), the highest one is at 10.25 c/d. These two peaks correspond to 1 c/d aliases of two of the three frequencies detected in photometric data (10.83 and 9.27 c/d respectively; Poretti 1991).

\(\langle v^2 \rangle\) (line rms width; bottom panel) has the highest peak at 13.20 c/d, which again is a 1 c/d alias of the highest amplitude photometric term (12.16 c/d).

The amplitudes of the first two moments for the three photometric modes as derived from the least squares fits of their time series are shown in Table 1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>(\langle v \rangle) (km/s)</th>
<th>(\langle v^2 \rangle) (km/s)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.27</td>
<td>0.39 ± 0.16</td>
<td>14 ± 17</td>
</tr>
<tr>
<td>12.16</td>
<td>0.19 ± 0.16</td>
<td>55 ± 17</td>
</tr>
<tr>
<td>10.83</td>
<td>0.52 ± 0.16</td>
<td>38 ± 17</td>
</tr>
</tbody>
</table>

As shown by Balona (1986b) the zero points of the 2nd and 4th moment (4135 ± 12 km/s)^2 and (37.0 ± 0.2) · 10^6 (km/s)^4 respectively) allow to estimate the projected rotational velocity and the intrinsic rms width of the lines.
We get $v \sin i = 134 \text{ km s}^{-1}$ and $W_i = 12 \text{ km s}^{-1}$. We see that there is an excellent agreement with the rotational velocity estimated from the fitting of the average line profiles.

3. Discussion and conclusions

The study of the line profile variations of HD 101158 shows the presence of two dominant high $|m|$ prograde modes with frequencies of 12.9 and 18.5 c/d and $m = -10$ and $-14$ or $-15$ respectively. The star has a quite high rotational velocity (132 km s$^{-1}$) and is seen almost equator on ($i \simeq 70–90^{\circ}$). Obviously these frequencies are in the observer’s reference frame; in the stellar corotating frame they are given by $\nu_{\text{obs}} + m/P_{\text{rot}}$, where the rotational period ($P_{\text{rot}}$) can be estimated from the stellar radius ($\sim 3R_\odot$) and rotational velocity as $1.1 \pm 0.1 \text{ d}$, thus we obtain $\sim 3.8 \text{ c/d}$ and $\sim 4.9$ or $5.8 \text{ c/d}$ for the two modes. Therefore it seems that the two perturbations have different propagation velocities even if they are of the same order. However given the uncertainties we cannot completely rule out the possibility that the propagation velocity is the same: for example for $m = -10$ and $-15$ this would require $P_{\text{rot}} = 0.9 \text{ d}$ and the corresponding frequency of the perturbations would be 1.8 c/d. In this case the ratio of the pulsation period to the rotation period is 62%, which means that the pulsation velocity could no longer be described in terms of one spherical harmonic (Lee & Sáio 1990, Aerts & Waëlkens 1993). In this case the mode identifications should be very questionable. This possibility is however not very likely: for example if the mode at 18.5 c/d has $m = -14$ the equal propagation velocity hypothesis would require $P_{\text{rot}} \approx 0.7 \text{ d}$ which can be ruled out because the rotational velocity would be close to the critical value.

In any case these frequencies are decidedly lower than those observed for the photometric terms (it has to be remembered

**Fig. 5.** Amplitudes (left panels) and phases (right panels) of the sine waves with $\nu = 18.50$ c/d which best fit the profile variations of the Ti II 4501 Å line (upper panels) and the Fe II 4508 Å line (lower panels)
that photometric terms have low degree modes so that their frequencies in the stellar corotating frame are not very different from the observed ones).

We have also seen that the frequency analysis of the first two moments give indications of the presence of the three modes detected photometrically in the spectroscopic data. Poretti (1991) tentatively identified these modes as the radial fundamental mode \( (\nu_3 = 9.27 \text{ c/d}) \), the radial first overtone mode \( (\nu_2 = 12.16 \text{ c/d}) \), and a non-radial \( p_1, \ell = 1 \) mode \( (\nu = 10.83 \text{ c/d}) \), however the fact that \( \nu_1 \) is probably present in \( \langle v_2 \rangle \) tends to exclude that this can be a radial mode since we expect that an axisymmetric mode does not show amplitude variations of the even moments with its frequency but only with even multiples of it (Balona 1986a, Aerts et al. 1992). It is quite strange that this, which was the photometric strongest term, is not apparent in the \( \langle v \rangle \) data.

High resolution spectroscopic studies of line profile variations of fast rotating \( \delta \) Scuti stars have shown that the presence of high azimuthal number non-radial modes is common among these objects (see for example Yang 1991) and the characteristics of these pulsations are quite similar: we still have perturbations moving from blue to red along the line profiles (prograde motions), with amplitudes of about 0.5% of the continuum and \( |m| \) of the order of 10. In some cases, when multiple high degree modes have been detected, it has been suggested that they could have the same frequency in the stellar corotating frame (e.g. Peg, Kennelly et al. 1992). We have seen that this possibility cannot be excluded for the two high degree modes of \( \delta \) Scuti star HD 101158. The presence of such high degree modes could be connected to the stellar rotational rate: for example no trace of these modes has been found in the \( \delta \) Scuti star X Cae, which has an intermediate rotational velocity \( (v_\text{e} \sin i = 70 \text{ km s}^{-1}) \) and is seen almost equator on, and instead possesses a pulsational
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Fig. 7. Observed (solid line) and computed (dashed line) profiles of the three observing nights for the 4502 Å Ti ii line. The small bar indicates the 3% of intensity with respect to the continuum. In each panel the time runs from the top to the bottom.

Fig. 8. First two moments of the line profiles.

Fig. 9. Frequency analysis of the first (upper two panels), and second (bottom panel) line moments.
spectrum with several low $\ell$,$|m|$ non-radial modes (Mantegass \& Poretti 1996).

Acknowledgements. I am grateful to Dr. L.A. Balona who kindly supplied his code for the computation of synthetic line profiles, to Dr. E. Poretti who stimulated this project and made a critical reading of the manuscript, and to Mr. J. Vialle, who checked the English form. Finally I want to express my thanks to the referee Dr. C. Aerts, whose observations and suggestions have led to a considerable improvement of the paper.

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